Non-Discrete P Systems

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1 Introduction

Until now, the usual variants of P systems have only a finite number of options in every step of their computations (that is, an associated computation tree is defined for them). In this way, irrespectively whether they are non-deterministic or probabilistic P systems, we obtain a discrete space of computations where the system evolves.

Here we present a variant of P systems which in every step can evolve in a non-discrete number of choices. For that, we do not use discrete multisets, but multisets where the multiplicity of the objects can be real (positive) numbers. The inspiration of this paper is that, in vitro, we can control neither the application of the rules nor the exact number of objects, but we deal with an approximate and non natural number of applications (possibly, related with the concentration of the objects in the membrane). In this way, the multiplicity of an object does not reflect the exact number of identical copies of it in the membrane, but its concentration in the solution.

2 Non-discrete Multisets

As it was used in [1], we can define a generalization of multisets by using non integer multiplicities.

Definition 2.1 *Let V be a finite alphabet. A* non-discrete multiset *(ND-multiset) over V is an application; $w : V \to \mathbb{R}^+$. We denote by $NDM(V)$ the set of non-discrete multisets over V.*

In a similar way to multisets, we can define the *support* of a ND-multiset ($supp(w)$), the usual operations between them:

- Addition: $(w_1 + w_2)(a) = w_1(a) + w_2(a)$.
- Subtraction: $(w_1 - w_2)(a) = w_1(a) - w_2(a)$ (it is not a total operation).
- Arithmetic subtraction: $(w_1 \boxminus w_2)(a) = \max\{w_1(a) - w_2(a), 0\}$.
- External product by real numbers: $(n \cdot w)(a) = n \cdot w(a)$.

and the usual relations:

- $w_1 \subseteq w_2$ $(w_1 \leq w_2) \iff \forall a \in V(w_1(a) \leq w_2(a))$.
- $w_1 \neq w_2 \iff \exists a \in V(w_1(a) \neq w_2(a))$.

And, finally, $\mathbf{0}$ stands for the empty multiset.

3 Non-discrete P systems

Now we formalize the variant of P systems that makes use of the non-discrete multisets. In this variant we allow neither the use of dissolutions nor active membranes (creation, duplication, charges, etc.), but we include in it the (now) *classic* transition P systems (where we can transform and move objects between adjacent membranes) and the communication ones (where we only can move objects taking into account the elements inside and immediately outside of the membrane).

In order to do that, we define the *ball* of a membrane as the set of membranes adjacent with it (and itself).

Definition 3.1 *Let μ be a membrane structure (a directed tree). For every node of μ, x, the* ball of x in μ *is the set $B_\mu(x) = \{y \in \mu : \ x \to y \vee y \to x \vee y = x\}$ (usually , we denote $B(x)$ instead of $B_\mu(x)$).*

In this context, a rule associated with a membrane, x, is a pair of applications, the first one indicating the objects consumed in every membrane adjacent with x, and the second one indicating the objects moved to every membrane adjacent with x.

Definition 3.2 *A rule, r, over a membrane structure, μ, is an application, $r : \mu \to NDM(V) \times NDM(V)$ (we will denote $r = (r_1, r_2)$).*

If $x \in \mu$ we will say that the rule r is associated with x if the following condition holds:

$$\forall y \notin B(x) \ (r(y) = (\mathbf{0}, \mathbf{0}))$$

We define a *non-discrete P system* as a membrane structure with a set of rules associated with each membrane.

Definition 3.3 *A* non-discrete P system *over an alphabet, V, is a pair $\Pi = (\mu, R)$ where μ is a membrane structure, and R is a set of rules over μ.*

A *cell* is defined by the allocation of a multiset to each membrane of the structure.

Definition 3.4 *A* cell *for Π is an application $C : \mu \to NDM(V)$. The set of cells for Π will be denoted by $Cell(\Pi)$.*

Starting from a cell, a *transition* is a, possibly, non-discrete application of the rules in a parallel manner.

Definition 3.5 *Let $\Pi = (\mu, R)$ be a non-discrete P system, and let C be a cell for Π, a* transition *for C is a non-discrete multiset over R, $T \in NDM(R)$, such that for every $x \in \mu$*

$$\sum_{r \in R} T(r) \cdot r_1(x) \subseteq C(x)$$

We will denote by $Tr(C)$ the set of transitions for C.

Now, the formalization of the application of the rules according with the selected transition can be given.

Definition 3.6 *Let* Π *be a non-discrete P system,* C *be a cell for* Π *and* $T \in Tr(C)$*. The cell obtained from* C *by the application of* T *is the cell,* $C' = T(C)$*, such that for every* $x \in \mu$*:*

$$C'(x) = C(x) + \sum_{r \in R} T(r) \cdot r_2(x) - \sum_{r \in R} T(r) \cdot r_1(x)$$

If we give an enumeration over the nodes of μ, $\{x_1, \ldots, x_j\}$, and the rules of R, $\{r^1, \ldots, r^N\}$, we can give a matrix form for the above transition:

$$[C'_1, \ldots, C'_j] = [C_1, \ldots, C_j] + [T_1, \ldots, T_N] \cdot \begin{bmatrix} r_2^1(x_1) - r_1^1(x_1) & \ldots & r_2^1(x_j) - r_1^1(x_j) \\ \vdots & \ddots & \vdots \\ r_2^N(x_1) - r_1^N(x_1) & \ldots & r_2^N(x_j) - r_1^N(x_j) \end{bmatrix}$$

where, C_i, C'_i, T_i stands for $C(x_i)$, $C'(x_i)$, $T(r_j)$, respectively. That will be written briefly as:

$$T(C) = C + T \cdot (R_2 - R_1)$$

and $T(C)(x) = C(x) + T \cdot (R_2 - R_1)(x)$.

4 Space of Transitions and Extremal Transitions

The set of *extremal transitions* is the set of transitions consuming the maximal amount of objects, in the following sense.

Definition 4.1 *The set of* extremal transitions *for* C *is the set of maximal points (regarding the above partial order) of* $Tr(C)$*, that is:*

$$ExTr(C) = \{T \in Tr(C) : \ \forall\ T' \in Tr(C)\ (T' \neq T \rightarrow \neg(T' \geq T))\}$$

That is, if we apply an extremal transition, then we can not apply more rules simultaneously over the remaining objects.

As we will see, a difference with the discrete case, in the non-discrete one we obtain that the set of transitions has good geometrical properties.

Lemma 4.1 *Let* Π *be a non-discrete P system. For every cell for* Π*,* C*, we obtain that its set of transitions,* $Tr(C)$*, is a convex and compact set.*

Proof.

Let C be a cell for Π. To see that $Tr(C)$ is a convex set, let $T, T' \in Tr(C)$ be two transitions for C, and let $p \in [0, 1]$, let us prove that $p \cdot T + (1 - p) \cdot T' \in Tr(C)$. It is direct to check that, for all $r \in R$, $p \cdot T(r) + (1 - p) \cdot T(r) \in \mathbb{R}^+$ holds.

Let $x \in \mu$:

$$\sum_{r \in R} (p \cdot T(r) + (1 - p) \cdot T'(r)) \cdot r_1(x) =$$

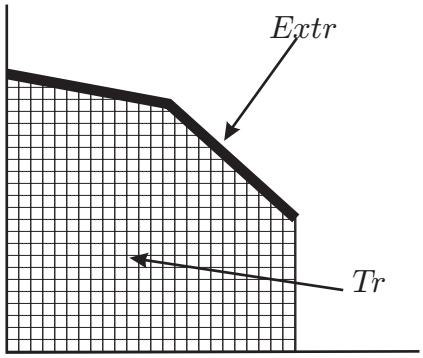
$$= p \cdot \sum_{r \in R} T(r) \cdot r_1(x) + (1 - p) \cdot \sum_{r \in R} T'(r) \cdot r_1(x) \subseteq$$

$$\subseteq p \cdot C(x) + (1 - p) \cdot C(x) = C(x)$$

It is easy to prove that $Tr(C)$ is compact, because it is a closed and bounded subset of some Euclidean space $\mathbb{R}^k$.

□

This is not true for $ExTr(C)$, that, in general, will not be a convex set, and trivial examples can be given. Of course, $Extr(C)$ remains to be a compact set.



5 Geometric aspects of non-discrete P systems

In this section we present some metrics in order to prove that, in case of finite computations (but not only in these ones), we can reduce ourselves to the study of non-discrete P systems where the multiplicities and number of applications are rational numbers.

We can consider that all above objects (non-discrete multisets, cells, transitions, sets of transitions, etc.) are subsets or applications in finite-dimensional Euclidean spaces, so all the metrics we define here will be usual.

Lemma 5.1 *Next applications are metrics in irrespective spaces:*

1. $d_{NDM(V)} : NDM(V) \times NDM(V) \longrightarrow \mathbb{R}^+$, *defined by*

$$d_{NDM(V)}(w_1, w_2) = \max\{|w_1(a) - w_2(a)| : \ a \in V\}$$

2. $d_C : Cell(C) \times Cell(C) \longrightarrow \mathbb{R}^+$, *defined by*

$$d_C(C, C') = \max\{d_{NDM(V)}(C(x), C'(x)) : \ x \in \mu\}$$

We denote by d_{Tr} the restriction of $d_{NDM(R)}$ to Tr.

By using this metric we can define something like a continuity in the application of the transitions. We can *control* the evolution of the system by taking *near* transitions:

Lemma 5.2 *Let Π be a non-discrete P system. There exist $N, K > 0$ (only depending on Π) such that for every C, cell for Π, and $T, T' \in Tr(C)$, if $d_{Tr}(T, T') < \varepsilon$, then*

$$d_C(T(C), T'(C)) < KN\ \varepsilon$$

Proof.

We take $N = card(R)$. Because of R is a finite set, there exists $K > 0$, such that:

$$\forall r \in R\ \forall a \in V\ (r_1(x)(a) \leq K\ \wedge\ r_2(x)(a) \leq K)$$

Let $x \in \mu$ and $a \in V$:

$$|T(C)(x)(a) - T'(C)(x)(a)| = |\sum_{r\in R}(T(r) - T'(r)) \cdot (r_2(x)(a) - r_1(x)(a))| \leq$$
$$\leq \sum_{r\in R} |T(r) - T'(r)| \cdot |r_2(x)(a) - r_1(x)(a)| < KN\ \varepsilon$$

From where we obtain: $d_C(T(C), T'(C)) < KN\ \varepsilon$.

□

Moreover, we can even prove something similar considering two different cells.

Lemma 5.3 *Let Π be a non-discrete P system, C, C' be two cells for Π, and T, T' be two transitions for C and C' respectively. Then*

$$d_C(T(C), T'(C')) \leq d_C(C, C') + KN \cdot d_{Tr}(T, T')$$

Proof.

Let $x \in \mu$, and $a \in V$:

$$|T(C)(x)(a) - T'(C')(x)(a)| =$$
$$= |C(x)(a) + \sum_{r\in R} T(r)(r_2(x) - r_1(x))(a) - C'(x)(a) - \sum_{r\in R} T'(r)(r_2(x) - r_1(x))(a)| \leq$$
$$\leq |C(x)(a) - C'(x)(a)| + |\sum_{r\in R}(T(r) - T'(r))(r_2(x) - r_1(x))(a)| \leq$$
$$\leq |C(x)(a) - C'(x)(a)| + \sum_{r\in R} |T(r) - T'(r)| \cdot |(r_2(x) - r_1(x))(a)| \leq$$
$$\leq d_C(C, C') + KN \cdot d_{Tr}(T, T')$$

□

We can go further and consider a metric between the sets of transitions.

Definition 5.1 *Let Π be a non-discrete P system, C, C' be two cells for Π. We define*

$$d(Tr(C), Tr(C')) = \max\{d(T, Tr(C')) :\ T \in Tr(C)\}$$

where $d(T, Tr(C')) = \min\{d_{Tr}(T, T') :\ T' \in Tr(C')\}$.

Lemma 5.4 *In this context, the application Tr is continuous. That is,*

$$\forall\ \varepsilon > 0\ \exists\ \delta > 0\ (d_C(C, C') < \delta \rightarrow d(Tr(C), Tr(C')) < \varepsilon)$$

Proof.

This lemma has a very technical proof, where its main idea is to consider the continuous dependence of the transitions from the content of the cells.

□

The combination of the two above lemmas allows us to obtain a general procedure for approximating in the evolution of the non-discrete P system.

Until now, what we can do is to approximate one step (and, of course, a finite computation) of the evolution of the non-discrete P system by another one where the transitions verify to be not so far form the original ones, and obtaining a *similar* final cell (in content). Of course, since the set of transitions are convex, this fact can be used in order to approximate computations by using only rational applications of rules. But, can we do the same if we consider only extremal transitions? The answer to this question is, in general, negative; nevertheless, if we put some restrictions in our P systems, we can give an affirmative answer.

Note 5.1 *If we restrict:*

- *the rules to be applications,* $r : \mu \to M(V) \times M(V)$*, where* $M(V)$ *stands for the usual* discrete *multisets;*
- *and we start from a* rational *cell (that is,* $\forall\ x \in \mu\ \forall\ a \in V\ (C(x)(a) \in \mathbb{Q})$*),*

then we can make approximations of extremal transitions by means of extremal transitions where all the values are rational. That is,

$$\forall\ \varepsilon > 0\ \forall\ T \in ExTr(C)\ \exists\ T' \in Extr(C) \cap \mathbb{Q}^N\ (d_{Tr}(T, T') < \varepsilon)$$

Of course, the application of a rational extremal transition over a rational cell provides a rational cell, so we can iterate this procedure along finite computations and obtain an approximation of all the computation by means of using only rational numbers.

6 Conclusions

We propose a new variant of P systems where the obtained space of computations is not a discrete space, but a dense one where in every step of the evolution a non finite (and dense) space of configurations can be produced.

This work is intended as an attempt to provide a new variant of P systems where only some approximate behaviors of the real reactions inside the cell are known. This approach is currently used in the development of a probabilistic software tool allowing the user to work with concentrations of the reactants, not with the exact number of each of them, trying to be nearer of the real case in laboratory.

But, also, this variant can provide new problems related with the computational power of the model. If we attend to the number of possible computations of a non-discrete P system, we realize that there is an uncountable amount of them, while in the discrete P systems this number is always countable. This raises the following question: how can we approximate the functioning of this devices by means of *classical* P systems?

Acknowledgment

Support for this research through the project TIC2002-04220-C03-01 of the Ministerio de Ciencia y Tecnología of Spain, cofinanced by FEDER funds, is gratefully acknowledged.

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