Tema 17
Sistema experto de cálculo simbólico

Introducción

- 1957: GPS (General Problem Solver): Newell, Simon y Shaw

- 1961: SAINT (Symbolic Automatic INTegrator): J. Slagle

- 1963: SINT (Symbolic INTegrator): J. Moses

- 1963: MAC (Machine Asisted Cognition)

- 1969: MACSYMA (MAC's SYmbolic Manipulation System): J. Moses

- 1996: Maxima 5.2: W. Schelter

Cálculos numéricos

• Aritmética

```
(C1) 2+3;
(D1)                              5
(C2) 7/6+9/10;

                                 31
(D2)                             --
                                 15
```

• Definición de variables

```
(C1) x;
(D1)                              X
(C2) x:3;
(D2)                              3
(C3) x^2;
(D3)                              9
(C4) kill(x);
(D4)                           DONE
(C5) x^2;

                                  2
(D5)                              X
```

• Operaciones múltiples

```
(C1) (a:2, b:3, a+b);
(D1)                              5
```

Cálculos numéricos

- ## Precisión

- ### Aritmética entera:

```
(C1) 2^100;
(D1)                         1267650600228229401496703205376
```

- ### Aritmética real:

```
(C1) sqrt(2);
(D1)                                    SQRT(2)
(C2) float(%);
(D2)                              1.4142135623730195B0
(C3) fpprec;
(D3)                                      16
(C4) fpprec:50$
(C5) bfloat(d1);
(D5)  1.4142135623730950488016887242096980785696718753770B0
```

- ## Sumas y productos generalizados

```
(C1) sum(x^n,n,1,5);
                          5     4     3     2
(D1)                     X  +  X  +  X  +  X  +  X
(C2) product(x+n*y,n,1,4);
(D2)             (Y + X) (2 Y + X) (3 Y + X) (4 Y + X)
(C3)
```

Algebra

• Definición de funciones

```
(C1) area(base,altura) := (base*altura)/2;

                                                BASE ALTURA
(D1)                         AREA(BASE, ALTURA) := -----------
                                                     2
(C2) area(4,7);
(D2)                                  14
(C3) area(2+3,2*3);
(D3)                                  15
(C4) area(x,2*y);
(D4)                                  X Y
```

• Polinomios

• Operaciones con polinomios:

```
(C1) (3+x)^2*(5*x-1)+8*(x^2-3*x+4)*(x-2);
                      2                       2
(D1)      8 (X - 2) (X  - 3 X + 4) + (X + 3)  (5 X - 1)
(C2) expand(%);
                        3          2
(D2)              13 X  - 11 X  + 119 X - 73
```

Algebra

- **División de polinomios:**

```
(C1) a : 2*x^3-3*x^2-5*x-5$
(C2) b : x-2$
(C3) quotient(a,b);
```

$$2 X^2 + X - 3$$

(D3)

```
(C4) remainder(a,b);
```

(D4) $- 11$

```
(C5) b*d3+d4;
```

(D5) $(X - 2) (2 X^2 + X - 3) - 11$

```
(C6) expand(%);
```

(D6) $2 X^3 - 3 X^2 - 5 X - 5$

- **Descomposición factorial de polinomios:**

```
(C1) (x-2)^2*(x+3);
```

(D1) $(X - 2)^2 (X + 3)$

```
(C2) expand(d1);
```

(D2) $X^3 - X^2 - 8 X + 12$

```
(C3) factor(d2);
```

(D3) $(X - 2)^2 (X + 3)$

Algebra

```
(C4) factor(y^4-x^4);
```

$$
\text{(D4)} \qquad\qquad (Y - X) \ (Y + X) \ (Y^2 + X^2)
$$

```
(C5) factor(x^(2*n)+2*x^n+1);
```

$$
\text{(D5)} \qquad\qquad (X^N + 1)^2
$$

```
(C6) factor(10!);
```

$$
\text{(D6)} \qquad\qquad 2^8 \ 3^4 \ 5^2 \ 7
$$

- **Máximo común divisor y mínimo común múltiplo:**

```
(C1) p : x^2-9$
(C2) q : x^2+6*x+9$
(C3) gcd(p,q);
```

$$
\text{(D3)} \qquad\qquad X + 3
$$

```
(C4) lcm(p,q);
```

$$
\text{(D4)} \qquad\qquad (X - 3) \ (X + 3)^2
$$

- **Valor numérico de un polinomio:**

```
(C1) at(x^2+y^3,[x=1,y=2]);
```

$$
\text{(D1)} \qquad\qquad 9
$$

Algebra

- **Fracciones algebraicas**

- Simplificación de fracciones algebraicas:

```
(C1) (b^2-a^2)/(b+a)^2;
```
```
                                2     2
                               B  -  A
(D1)                           --------
                                        2
                               (B + A)
```
```
(C2) ratsimp(%);
```
```
                                B - A
(D2)                            -----
                                B + A
```

- Operaciones con fracciones algebraicas:

```
(C1) (1/x-1/y)*(1/x+1/y)*(1+(x+y)/(x-y))*(2+2*y/(x-y));
              1   1   1   1    2 Y          Y + X
(D1)         (- - -) (- + -) (----- + 2) (----- + 1)
              X   Y   Y   X   X - Y        X - Y
(C2) ratsimp(%);
                               4 Y + 4 X
(D2)                           ---------
                                3       2
                               Y  - X Y
```

Algebra

- **Valor numérico de una fracción algebraica:**

```
(C1) (3*a-2*b^2)/(3*a*b);
```

```
                                          2
                                 3 A - 2 B
(D1)                             ----------
                                    3 A B
```

```
(C2) at(%,[a=-2,b=3]);
```

```
                                     4
(D2)                                 -
                                     3
```

- **Verdadero valor de una fracción:**

```
(C1) (x^2-1)/(x-1);
```

```
                                   2
                                  X  - 1
(D1)                              ------
                                  X - 1
```

```
(C2) at(%,x=1);
Division by 0
(C3) at(ratsimp(d1),x=1);
(D3)                                 2
```

Algebra

- **Ecuaciones y sistemas**

- **Ecuaciones:**

```
(C1) e1:x^4-(a^2+b^2)*x^2+a^2*b^2=0;
```
$$X^4 - (B^2 + A^2) X^2 + A^2 B^2 = 0$$
```
(D1)
(C2) solve(e1,x);
```
$$[X = - A, X = A, X = - B, X = B]$$
```
(D2)
(C3) e2:sqrt(3*x-2)-4=0;
```
$$SQRT(3 X - 2) - 4 = 0$$
```
(D3)
(C4) solve(e2);
```
$$[X = 6]$$
```
(D4)
(C5) e3:x^3-2*x^2+x-2=0;
```
$$X^3 - 2 X^2 + X - 2 = 0$$
```
(D5)
(C6) solve(e3);
```
$$[X = - \%I, X = \%I, X = 2]$$
```
(D6)
(C7) solve(sin(x)*cos(x)=0,x);
```

```
SOLVE is using arc-trig functions to get a solution.
Some solutions will be lost.
```
$$[X = 0, X = \frac{\%PI}{2}]$$
```
(D7)
```

Algebra

- **Sistemas:**

```
(C1)  e1:x+y=8$
(C2)  e2:x*y=12$
(C3)  solve([e1,e2],[x,y]);
(D3)                          [[X = 2, Y = 6], [X = 6, Y = 2]]
(C4)  solve([x^2=5,x=5],x);
(D4)                                      []
```

Algebra lineal

- **Definición de matrices**

- **Matrices generales:**

```
(C1) a:matrix([4,0,6],[1,6,2],[3,9,0]);
```

```
                              [ 4   0   6 ]
                              [           ]
(D1)                          [ 1   6   2 ]
                              [           ]
                              [ 3   9   0 ]
```

```
(C2) a[3,2];
(D2)                               9
(C3) row(a,3);
(D3)                          [ 3   9   0 ]
(C4) col(a,2);
```

```
                                 [ 0 ]
                                 [   ]
(D4)                             [ 6 ]
                                 [   ]
                                 [ 9 ]
```

Algebra lineal

- **Matrices especiales:**

```
(C1) zeromatrix(2,3);
                                    [ 0   0   0 ]
(D1)                                [           ]
                                    [ 0   0   0 ]

(C2) ident(2);
                                      [ 1   0 ]
(D2)                                  [       ]
                                      [ 0   1 ]

(C3) diagmatrix(2,a);
                                      [ A   0 ]
(D3)                                  [       ]
                                      [ 0   A ]
```

- **Definición de matrices mediante funciones:**

```
(C1) f[i,j] := 10*i+j$
(C2) m : genmatrix(f,2,3)$
(C3) m;
                                  [ 11   12   13 ]
(D3)                              [              ]
                                  [ 21   22   23 ]
```

Algebra lineal

- **Definición reducida de matrices:**

```
(C1) a:entermatrix(3,3);

Is the matrix  1. Diagonal  2. Symmetric  3. Antisymmetric
               4. General
Answer 1, 2, 3 or 4 : 1;
Row 1 Column 1:  x;
Row 2 Column 2:  y;
Row 3 Column 3:  z;


Matrix entered.
                              [ X  0  0 ]
                              [         ]
(D1)                          [ 0  Y  0 ]
                              [         ]
                              [ 0  0  Z ]

(C2) a;
                              [ X  0  0 ]
                              [         ]
(D2)                          [ 0  Y  0 ]
                              [         ]
                              [ 0  0  Z ]
```

Algebra lineal

- **Operaciones con matrices:**

(C1) A : matrix([9,8],[7,6]);

```
                                    [  9   8  ]
(D1)                                [         ]
                                    [  7   6  ]
```

(C2) B : matrix([0,1],[3,2]);

```
                                    [  0   1  ]
(D2)                                [         ]
                                    [  3   2  ]
```

(C3) A + B;

```
                                    [  9    9  ]
(D3)                                [          ]
                                    [ 10    8  ]
```

(C4) A - B;

```
                                    [  9   7  ]
(D4)                                [         ]
                                    [  4   4  ]
```

(C5) A . B;

```
                                    [ 24   25 ]
(D5)                                [         ]
                                    [ 18   19 ]
```

Algebra lineal

- **Matriz traspuesta:**

```
(C1) A : matrix([5,3,6],[2,0,7],[8,1,0])$
(C2) transpose(a);
```

```
                          [ 5   2   8 ]
                          [           ]
(D2)                      [ 3   0   1 ]
                          [           ]
                          [ 6   7   0 ]
```

- **Matriz inversa:**

```
(C1) A : matrix([-4,9],[1,-2])$
(C2) invert(A);
```

```
                          [ 2   9 ]
(D2)                      [       ]
                          [ 1   4 ]
```

```
(C3) A . %;
```

```
                          [ 1   0 ]
(D3)                      [       ]
                          [ 0   1 ]
```

Algebra lineal

- **Potencia de matrices:**

```
(C1) A : matrix([1,1],[0,1]);
```
```
                              [ 1   1 ]
(D1)                          [       ]
                              [ 0   1 ]
```
```
(C2) A^^2;
```
```
                              [ 1   2 ]
(D2)                          [       ]
                              [ 0   1 ]
```
```
(C3) A^^3;
```
```
                              [ 1   3 ]
(D3)                          [       ]
                              [ 0   1 ]
```
```
(C4) A^^-1;
```
```
                              [ 1  - 1 ]
(D4)                          [        ]
                              [ 0   1  ]
```
```
(C5) invert(A);
```
```
                              [ 1  - 1 ]
(D5)                          [        ]
                              [ 0   1  ]
```

Algebra lineal

- **Determinante:**

```
(C1) A : matrix([1,1,1],[x,y,z],[y+z,z+x,x+y]);
                              [   1      1      1   ]
                              [                     ]
(D1)                          [   X      Y      Z   ]
                              [                     ]
                              [ Z + Y  Z + X  Y + X ]
(C2) determinant(A);
(D2)            Z(Z+Y)-Y(Z+Y)-Z(Z+X)+X(Z+X)+Y(Y+X)-X(Y+X)
(C3) expand(%);
(D3)                                   0
```

- **Rango:**

```
(C1) A : matrix([-1,1,-2],[1,1,0],[2,1,1]);
                              [ - 1   1   - 2 ]
                              [               ]
(D1)                          [   1   1    0  ]
                              [               ]
                              [   2   1    1  ]
(C2) rank(A);
(D2)                                   2
```

Cálculo

- ## Límites:

```
(C1) limit((x^2-4)/(x-2),x,2);
(D1)                                    4
(C2) limit((x^n-a^n)/(x-a),x,a);
                                        N - 1
(D2)                                   A      N
(C3) limit((x^(1/n)-1)/(x^(1/m)-1),x,1);
                                        M
(D3)                                    -
                                        N
(C4) limit(1/(x-1)^4,x,1);
(D4)                                   INF
(C5) limit(-1/(x-1)^4,x,1);
(D5)                                  MINF
(C6) limit((4*x+3)/(2*x-1),x,inf);
(D6)                                    2
(C7) limit((1+2/x)^x,x,inf);
                                        2
(D7)                                  %E
(C8) limit((cos(x))^(1/x),x,0);
(D8)                                    1
```

Cálculo

- **Límites laterales:**

```
(C9) f(x) := (2*x+abs(x))/(4*x-3*abs(x));

                                 2 X + ABS(X)
(D9)                     F(X) := --------------
                                 4 X - 3 ABS(X)
(C10) limit(f(x),x,0);

                          ABS(X)              2 X
(D10)             limit  ------------- + --------------
                  X -> 0 4 X - 3 ABS(X)   4 X - 3 ABS(X)
(C11) limit(f(x),x,0,minus);

                                  1
(D11)                             -
                                  7
(C12) limit(f(x),x,0,plus);
(D12)                             3
```

Cálculo

• Derivadas:

```
(C1) diff((sin(4*x))^3,x);
```

$$(D1) \qquad\qquad 12 \; COS(4\;X) \; SIN^2(4\;X)$$

```
(C2) diff(x^4+3*x^2-5,x,2);
```

$$(D2) \qquad\qquad 12 \; X^2 + 6$$

```
(C3) diff(x^y,x);
```

$$(D3) \qquad\qquad X^{Y-1} \; Y$$

```
(C4) diff(x^y,y);
```

$$(D4) \qquad\qquad X^Y \; LOG(X)$$

• Fórmula de Taylor:

```
(C1) taylor(%e^x,x,0,5);
```

$$(D1)/T/ \qquad 1 + X + \frac{X^2}{2} + \frac{X^3}{6} + \frac{X^4}{24} + \frac{X^5}{120} + \; . \; . \; .$$

```
(C2) taylor(x^3-4*x^2+7,x,1,3);
```

$$(D2)/T/ \qquad 4 - 5(X-1) - (X^2 - 1) + (X - 1)^3 + \; . \; . \; .$$

Cálculo

- **Integrales indefinidas:**

```
(C1)  integrate(1/x,x);
(D1)                                      LOG(X)
(C2)  integrate(1/(x*log(x)),x);
(D2)                                   LOG(LOG(X))
(C3)  integrate(sin(3*x)*(cos(3*x))^4,x);
                                          5
                                     COS (3 X)
(D3)                               - ---------
                                         15
(C4)  integrate(x*%e^x,x);
                                               X
(D4)                               (X - 1) %E
(C5)  integrate(1/(x^2+a^2),x);
                                          X
                                     ATAN(-)
                                          A
(D5)                               -------
                                       A
```

Cálculo

• Integrales definidas:

```
(C1) integrate(3*x^2,x,1,2);
(D1)                                 7
(C2) f(x) := integrate(1/log(t),t,1,x)$
(C3) f(y);
Is  Y - 1  positive, negative, or zero?
pos;

                                Y
                                /
                                [      1
(D3)                            I   ------ dT
                                ]  LOG(T)
                                /
                                 1
(C4) diff(f(x),x);
Is  X - 1  positive, negative, or zero?
pos;

                                   1
(D4)                             ------
                                 LOG(X)
(C5) integrate(1/x^2,x,-1,1);
Integral is divergent
(C6) integrate(1/(1+x^2),x,0,inf);

                                  %PI
(D6)                              ---
                                   2
```

Cálculo

• Integración numérica:

```
(C1) showtime : true$
Evaluation took 0.00 seconds (0.64 elapsed)
(C2) integrate(1/x,x,1,2);
Evaluation took 0.68 seconds (3.08 elapsed)
(D2)                                     LOG(2)
(C3) float(d2);
Evaluation took 0.00 seconds (00.70 elapsed)
(D3)                          0.69314718055994529
(C4) romberg(1/x,x,1,2);
Evaluation took 00.10 seconds (00.20 elapsed)
(D4)                          0.69314790148123473
(C5) integrate(1/log(x),x,2,4);
Evaluation took 0.45 seconds (1.09 elapsed)
                                   4
                                   /
                                   [      1
(D5)                               I   ------ dX
                                   ]   LOG(X)
                                   /
                                    2
(C6) romberg(1/log(x),x,2,4);
Evaluation took 00.30 seconds (0.30 elapsed)
(D6)                          1.9224216144693915
```

Cálculo

- **Sumatorios:**

(C1) sum(i^2,i,1,n);

```
                                        ====
                                        \         2
(D1)                                     >        I
                                        /
                                        ====
                                        I = 1
```

(C2) simpsum:true;

(D2)   TRUE

(C3) sum(i^2,i,1,n);

```
                                          3         2
                                        2 N   + 3 N   + N
(D3)                                    ---------------
                                               6
```

(C4) sum(x^k,k,0,n);

```
                                         N + 1
                                        X       - 1
(D4)                                    ----------
                                          X - 1
```

Cálculo

• Series numéricas:

```
(C1) sum((-1/4)^n,n,1,inf);

                                        1
(D1)                                  - -
                                        5
(C2) sum(r^n,n,0,inf);
Is  ABS(R) - 1  positive, negative, or zero?


neg;

                                        1
(D2)                                  -----

                                      1 - R
(C3) sum(1/n,n,1,inf);
(D3)                                   INF
(C4) sum((-1)^n,n,1,inf);
(D4)                                UNDEFINED
```

Cálculo

- **Ecuaciones diferenciales:**

```
(C1) e1 : ´diff(f(x),x)=3*x^2+2*x;
                             d                 2
(D1)                        -- (F(X)) = 3 X  + 2 X
                             dX
(C2) desolve(e1,f(x));
                                 3     2
(D2)                        F(X) = X  + X  + F(0)
(C3) e2 : ´diff(y(x),x)=a*y(x);
                             d
(D3)                        -- (Y(X)) = A Y(X)
                             dX

(C4) desolve(e1,y(x));
                                            A X
(D4)                        Y(X) = Y(0) %E
(C3) atvalue(y(x),x=0,1)$
(C5) desolve(e,y(x));
                                            A X
(D5)                        Y(X) = %E
```

Programación

- **Iteración:**

```
(C1) for i from 1 thru 3 do print("I =",i);
I = 1
I = 2
I = 3
(D1)                              DONE
(C2) for i from 1 step 2 thru 3 do print("I =",i);
I = 1
I = 3
(D2)                              DONE
(C3) i:1$
(C4) for i while i<=2 do print("I =",i);
I = 1
I = 2
(D4)                              DONE
(C5) i;
(D5)                                 1
(C6) for i unless i>2 do print("I =",i);
I = 1
I = 2
(D6)                              DONE
(C7)  for i in [1,2] do print("I =",i);
I = 1
I = 2
(D7)                              DONE
```

Programación

- **Condicionales:**

```
(C1) fib(n) := if n=1 or n=2
                then 1
                else fib(n-1)+fib(n-2)$
(C2) fib(6);
(D2)                                    8
```

- **Definiciones recursivas mediante índices:**

```
(C1) fact[0] : 1$
(C2) fact[n] := n*fact[n-1]$
(C3) fact[3];
(D3)                                    6
(C4) arrayinfo(fact);
(D4)                    [HASHED, 1, [0], [1], [2], [3]]
```

- **Booleanos:**

```
(C1) is(9 > 2+3);
(D1)                                    TRUE
(C2) is(9 > 2+3 and 2^3 = 2*3);
(D2)                                    FALSE
(C3) is(9 > 2+3 or 2^3 = 2*3);
(D3)                                    TRUE
(C4) x:9$
(C5) is(x=8+1);
(D5)                                    TRUE
```

Programación

- **Bloques y listas:**

```
(C1) es_primo(n) := is(factor(n) = n)$
(C2) es_primo(7);
(D2)                                    TRUE
(C3) es_primo(6);
(D3)                                    FALSE
(C4) primos(n,m) :=
       for i from n thru m
           do if es_primo(i) then print(i)$
(C5) primos(10,30);
11
13
17
19
23
29
(D5)                                    DONE
```

Programación

```
(C6) lista_de_primos(n,m) :=
        (res:[],
          for i from n thru m
              do if es_primo(i)
                    then res:cons(i,res),
          reverse(res))$
(C7) lista_de_primos(10,30);
(D7)                            [11, 13, 17, 19, 23, 29]
(C8) res;
(D8)                            [29, 23, 19, 17, 13, 11]
(C9) kill(res);
(D9)                                         DONE
(C10) lista_de_primos(n,m) := block([res],
          res:[],
          for i from n thru m
              do if es_primo(i)
                    then res:cons(i,res),
          reverse(res))$
(C11) lista_de_primos(10,30);
(D11)                           [11, 13, 17, 19, 23, 29]
(C12) res;
(D12)                                        RES
```

Programación

- ## Método de Newton:

$$f(x) = 0$$

$$x_0 = a, \; x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

```
(C1) newton(f,aprox) := block([numer],
        numer:true,
        do (if abs(f(aprox)) < 0.001 then return(aprox),
            aprox:at(x-f(x)/diff(f(x),x),x=aprox)))$
(C2) g(x) := x^2-3$
(C3) newton(g,10);
(D3)                            1.7320982711195376
(C4) h(x) := 2*x-log(x)-4$
(C5) newton(h,10);
(C5) newton(h,10);
(D5)                            2.4475434805898919
(C6) h(d5);
(D6)                            2.1006076442731114E-6
```