Ejercicio 7.1. Probar mediante deducción natural:

1. $\forall x.(P(x) \to Q(x)) \vdash \forall x.P(x) \to \forall x.Q(x)$
2. $\exists x.\neg P(x) \vdash \neg\forall x.P(x)$
3. $\forall x.P(x) \vdash \forall y.P(y)$
4. $\forall x.(P(x) \to Q(x)) \vdash (\forall x.\neg Q(x)) \to (\forall x.\neg P(x))$
5. $\forall x.(P(x) \to \neg Q(x)) \vdash \neg(\exists x.(P(x) \land Q(x)))$
6. $\forall x.\forall y.P(x,y) \vdash \forall u.\forall v.P(u,v)$
7. $\exists x.\exists y.P(x,y) \vdash \exists u.\exists v.P(u,v)$
8. $\exists x.\forall y.P(x,y) \vdash \forall y.\exists x.P(x,y)$
9. $\exists x.(P(a) \to Q(x)) \vdash P(a) \to \exists x.Q(x)$
10. $P(a) \to \exists x.Q(x), \text{actual } i \vdash \exists x.(P(a) \to Q(x))$
11. $\exists x.P(x) \to Q(a) \vdash \forall x.(P(x) \to Q(a))$
12. $\forall x.(P(x) \to Q(a)), \text{actual } i \vdash \exists x.(P(x) \to Q(a))$
13. $\forall x.P(x) \lor \forall x.Q(x) \vdash \forall x.(P(x) \lor Q(x))$
14. $\exists x.(P(x) \land Q(x)) \vdash \exists x.P(x) \land \exists x.Q(x)$
15. $\forall x.\forall y.(P(y) \to Q(x)) \vdash \exists y.P(y) \to \forall x.Q(x)$
16. $\neg\forall x.\neg P(x), \text{actual } j \vdash \exists x.P(x)$
17. $\forall x.\neg P(x) \vdash \neg\exists x.P(x)$
18. $\exists x.P(x) \vdash \neg\forall x.\neg P(x)$
19. $P(a) \to \forall x.Q(x) \vdash \forall x.(P(a) \to Q(x))$
20. $\forall x.\forall y.\forall z.(R(x,y) \land R(y,z) \to R(x,z)), \forall x.\neg R(x,x) \vdash \forall x.\forall y.(R(x,y) \to \neg R(y,x))$
21. $\forall x.(P(x) \lor Q(x)), \exists x.\neg Q(x), \forall x.(R(x) \to \neg P(x)) \vdash \exists x.\neg R(x)$
22. $\forall x.(P(x) \to (Q(x) \lor R(x))), \neg\exists x.(P(x) \land R(x)) \vdash \forall x.(P(x) \to Q(x))$
23. $\exists x.\exists y.(R(x,y) \lor R(y,x)) \vdash \exists x.\exists y.R(x,y)$

Ejercicio 7.2. Probar mediante deducción natural:

1. $P(c), (\forall x)(P(x) \to \neg Q(x)) \vdash \neg Q(c)$.
2. $(\forall x)(P(x) \to Q(x)), (\forall x)P(x) \vdash (\forall x)Q(x)$
3. $(\forall x)P(x) \vdash (\exists x)P(x)$
4. $(\forall x)(P(x) \to Q(x)), (\exists x)P(x) \vdash (\exists x)Q(x)$

5. $(\forall x)(Q(x) \to R(x)), (\exists x)(P(x) \land Q(x)) \vdash (\exists x)(P(x) \land R(x))$
6. $(\exists x)P(x), (\forall x)(\forall y)(P(x) \to Q(y)) \vdash (\forall y)Q(y)$
7. $\neg(\forall x)F \equiv (\exists x)\neg F$
8. $(\forall x)(F \land G) \equiv (\forall x)F \land (\forall x)G$
9. $(\exists x)F \lor (\exists x)G \equiv (\exists x)(F \lor G)$
10. $(\exists x)(\exists y)F \equiv (\exists y)(\exists x)F$

Nota: En las transparencias del tema se encuentra una solución.

Ejercicio 7.3. (*Primer parcial del 2005–06 (Grupo 1)*) Decidir, mediante deducción natural, si

$$\{(\forall x)[R(x) \to Q(x)], (\exists x)[P(x) \land \neg Q(x)]\} \models (\exists x)[P(x) \land \neg R(x)].$$

Ejercicio 7.4. (*Primer parcial del 2005–06 (Grupo 1)*) Demostrar por deducción natural con Jape

1. $\exists x.(p(x) \land q(x)), \forall y.(p(y) \to r(y)) \vdash \exists x.(r(x) \land q(x))$
2. $\forall x.r(x,x), \forall x.\forall y.\forall z.(\neg r(x,y) \land \neg r(y,z) \to \neg r(x,z)) \vdash \forall x.\forall y.(r(x,y) \lor r(y,x))$
3. $\exists x.\exists y.(R(x,y) \lor R(y,x)) \vdash \exists x.\exists y.R(x,y)$
4. $\forall x.(p(x) \to \exists y.q(y)), \text{actual } i \vdash \forall x.\exists y.(p(x) \to q(y))$

Ejercicio 7.5. (*Primer parcial del 2005–06 (Grupo 2)*) Decidir, mediante deducción natural, si

$$\{(\forall x)[P(x) \to \neg C(x)], (\exists x)[C(x) \land B(x)]\} \models (\exists x)[B(x) \land \neg P(x)]$$

Ejercicio 7.6. (*Primer parcial del 2005–06 (Grupo 2)*) Decidir, mediante tableros semánticos, si

$$\neg(\exists x)P(x) \models (\forall y)[((\exists z)P(z)) \to P(y)].$$

Ejercicio 7.7. (*Primer parcial del 2005–06 (Grupo 2)*) Demostrar por deducción natural con Jape

1. $\forall x.\exists y.(p(x) \to q(y)) \vdash \forall x.(p(x) \to \exists y.q(y))$
2. $\neg\forall x.(p(x) \to q(a)) \vdash \exists x.p(x) \land \neg q(a)$
3. $\forall x.p(x), \forall x.(p(x) \to q(x) \lor r(x)), \exists x.\neg q(x) \vdash \exists x.r(x)$
4. $\forall x.\forall y.(r(x,y) \to r(y,x)), \forall x.\forall y.(r(x,y) \lor r(y,x)) \vdash \forall x.\forall y.\forall z.(\neg r(x,y) \land \neg r(y,z) \to \neg r(x,z))$

Ejercicio 7.8. (*Examen de Junio de 2005*) Probar mediante deducción natural:

1. $\neg(\forall x)P(x) \vdash (\exists x)\neg P(x)$
2. $\{(\forall x)(\forall y)[((\exists z)R(y,z)) \to R(x,y)],\ (\exists x)(\exists y)R(x,y)\} \ \vdash\ (\forall x)(\forall y)R(x,y)$

Ejercicio 7.9. (*Examen de Septiembre de 2005*) Probar mediante deducción natural:

1. $\exists x.(P(x) \land \neg Q(x)) \to \forall y.(P(y) \to R(y)), \exists x.(P(x) \land S(x)), \forall x.(P(x) \to \neg R(x)) \vdash \exists x.(S(x) \land Q(x))$
2. $\vdash \neg\exists x.\forall y.(P(y,x) \leftrightarrow \neg P(y,y))$

Ejercicio 7.10. (*Examen de Diciembre de 2005*) Probar mediante deducción natural usando Jape:

1. $\forall x.(\exists y.R(x,y) \to \exists y.(\forall z.R(y,z) \land R(x,y))), \exists x.\exists y.R(x,y) \vdash \exists x.\forall y.R(x,y)$
2. $\forall x.(P(x) \to \forall y.(Q(y) \to R(x,y))), \exists x.(P(x) \land \exists y.\neg R(x,y)) \vdash \neg\forall x.Q(x)$