A Membrane Computing Model for Ballistic Depositions

Carmen Graciani-Díaz, Miguel A. Gutiérrez-Naranjo, Mario J. Pérez-Jiménez

Research Group on Natural Computing
University of Sevilla
Avda Reina Mercedes s/n, 41012 Sevilla, Spain
{cgdiaz,magutier,marper}@us.es

Summary. Ballistic Deposition was proposed by Vold [9] and Sutherland [8] as a model for colloidal aggregation. These early works were later extended to simulate the process of vapor deposition. In general, Ballistic Deposition models involve $(d+1)$-dimensional particles which rain down sequentially at random onto a d-dimensional substrate; when a particle arrives on the existing agglomeration of deposited particles, it sticks to the first particle it contacts, which may result in lateral growth. In this paper we present a first P system model for Ballistic Deposition with $d = 1$.

1 Introduction

Some recent discoveries on the dynamical process of surface growth have encouraged the scientific community to revisit the study of systems exhibiting rough interfaces. In Nature, there exist many examples of rough interfaces, actually, *all* surfaces in Nature can be seen as rough surfaces, since the concept of roughness is associated to the scale of observation and surfaces on Nature are far from be smooth if observed at appropriate scale.

The propagation of forest fires [5], the growth of a colony of bacteria [3] or the propagation of reaction fronts in catalyzed reactions [1] are real-world examples where the *frontier* between two media are far from being smooth. In this cases, the interfaces can be hardly modeled with Euclidean geometry and it is necessary to consider new tools in order to handle them. Moreover, in that cases, we are interested not only in the morphology of the interfaces from a static point of view, but in the dynamics of how the interface develops in time.

These dynamics can be studied from two complementary approaches:

- *Discrete approaches* where the position of each particle of the surface is well defined. This approach is getting more consideration at the atomic level in the last years due to the use of new technology as the scanning tunneling microscopy, capable of identifying not only the structure of the lattice of particles, but the position of individual atoms as well.

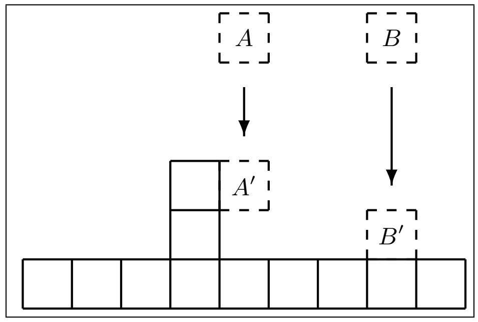

Fig. 1. Ballistic Deposition

- *Continuum approaches* view the surface on a *coarse-grained* scale, in which every property is averaged over a small volume containing many atoms. Their predictive power is limited to length scales larger than the typical inter-atom distance.

This paper is devoted to the study of a process of formation of rough surfaces called *Ballistic Deposition* (BD). To this aim, we will explore the capability of some Membrane Computing devices as tools for modeling BD in a discrete approach.

The paper is organized as follows: first the Ballistic Deposition model is briefly described. In Section 3, *deposition P systems* are presented and following this model, a P system simulating the dynamics of Ballistic Deposition is presented in Section 4. Some conclusions are presented in Section 5. The paper ends with the Bibliography and an Appendix with the proof of our main result.

2 Ballistic Deposition

In Nature, some interfaces are formed as result of a deposition process, other shrink due to erosion. A typical example of deposition process is the random fall of snowflakes on the ground floor. The randomness in the deposition process leads to a rough surface.

There exist many deposition models which try to represent different natural process. The simplest way to define such models is on a lattice where particles are deposited onto a surface oriented perpendicular to the particle trajectories, but other versions have been also investigated[1].

Ballistic Deposition (BD) was proposed by Vold [9] and Sutherland [8] as a model for colloidal aggregation. These early works were later extended to simulate the process of vapor deposition. In this model, a particle is released from a position above the surface. The particle follows a straight vertical trajectory until it reaches the surface, whereupon it sticks (see Figure 1).

[1] A good starting point for the study of depositions is [2].

In general, Ballistic Deposition models involve $(d+1)$-dimensional particles which rain down sequentially at random onto a d-dimensional substrate; when a particle arrives on the existing agglomeration of deposited particles, it sticks to the first particle it contacts, which may result in lateral growth. Many mathematical models exist in order to describe Ballistic Depositions. Here we follow M.D. Penrose in [6], where all particles are assumed identical is presented.

In Penrose's mathematical model, the substrate is $\mathbb{R}^d \times \{0\}$, identified with $\mathbb{R}^d$ or some subregion thereof. All particles are $(d+1)$-dimensional solids. Particles arrive sequentially at random positions in $\mathbb{R}^d$. When a particle arrives at a position $x \in \mathbb{R}^d$, it slides down the ray $\{x\} \times [0, \infty)$ until the particle hits a position adjacent to either the substrate or a previously deposited particle where is permanently fixed. The difference between lattice and continuum models is that in the lattice model the positions at which particle arrive are restricted to be in the integer lattice $\mathbb{Z}^d$.

Let $\mathbf{0}$ denote the origin in $\mathbb{Z}^d$. A *displacement function* is a mapping $D : \mathbb{Z}^d \to [-\infty, \infty)$ verifying:

- $D(\mathbf{0}) = 1$
- The set $\mathcal{N} = \{x \in \mathbb{Z}^d \,:\, D(x) \neq -\infty\}$ is finite but has at least two elements (one of which is the origin)

For $z \in \mathbb{Z}^d$, let $\mathcal{N}_x = \{x + y \,:\, y \in \mathcal{N}\}$ and $\mathcal{N}^*_x = \{x - y \,:\, y \in \mathcal{N}\}$. The set $\mathcal{N}$ is a neighborhood of the origin and $\mathcal{N}_x$ is a neighborhood of x. The idea of a displacement function is that if a particle arrives at $y \in \mathcal{N}_x$ then it cannot slide down the ray $y \times [0, +\infty)$ below the position at height $D(y - x)$. In this way, if $h(x, t)$ measures the height of the interface at site x at time t then

$$h(x, t+1) = max\{h(y, t) + D(x - y) \,:\, y \in \mathbb{Z}^d\}$$

since $-\infty + x = -\infty$ for all $x \in \mathbb{R}$ then

$$h(x, t+1) = max\{h(y, t) + D(x - y) \,:\, y \in \mathcal{N}^*_x\}$$

In this paper we follow the version of ballistic deposition considered in [7], the nearest neighbor model, where $\mathcal{N} = \{z \in \mathbb{Z}^d \,:\, \|z\|_1 \leq 1\}$ and the displacement function D is given by $D(x) = 0$ for $x \in \mathcal{N} - \{\mathbf{0}\}$. We are considering that the dimension of the substrate is $d = 1$ and therefore

$$\begin{aligned}
h(x, t+1) &= max\{h(y, t) + D(x - y) \,:\, y \in \mathcal{N}^*_x\} \\
&= max\{h(y, t) + D(x - y) \,:\, y \in \{x - 1, x, x + 1\}\} \\
&= max\{h(x - 1, t) + D(1), h(x, t) + D(0), h(x + 1, t) + D(-1)\} \\
&= max\{h(x - 1, t) + 0, h(x, t) + 1, h(x + 1, t) + 0\} \\
&= max\{h(x - 1, t), h(x, t) + 1, h(x + 1, t)\}
\end{aligned}$$

3 P systems

The chosen P systems model can be considered a subclass of tissue-like P system, since we do not consider membranes surrounding other ones, but a sequence of cells linked by communication channels. The intuition behind this structure is that each cell represent a column of the aggregate and the pieces of information needed for encoding the growth process are encoded on the multisets of objects in the cells.

In this model we use two very powerful membrane computing tools: the cooperation and the use of polarizations of the cells. Both features allow us an efficient design of P systems in order to perform the simulation. The study of minimal resources, i.e., to know whether the deposition process can be simulated without some of the used ingredients falls out of the scope of this paper.

Formally, a *deposition P system* of degree L is a construct of the form

$$\Pi = (O, \mu, env, v_1, \ldots, v_L, v_{env}, P, R)$$

where:

1. O is the alphabet of objects;
2. μ is a *cell structure* consisting of L cells bijectively labelled with $\{1, \ldots, L\}$. For all $i \in \{1, \ldots, L-1\}$ there exist an edge between the cell i and the cell $i+1$. We will also consider an edge between the cell L and the cell 1. For the sake of simplicity, we will identify the indices $L+1$ and 1; also, if a cell has polarization 0, we will omit the symbol 0.
3. env is the environment. It represents the region surrounding the cell structure μ. Some objects can be also placed in this region.
4. $v_1, \ldots, v_L, v_{env}$ are strings over O, describing the *multisets of objects* placed in the corresponding cells of μ or in the environment.
5. $P = \{0, +, -\}$ is the set of polarizations.
6. R is a finite set of *rules*, of the following forms:
   a) $[v_1 \to v_2]_i^e$ where $i \in \{1, \ldots, L\}$, $e \in P$ and v_1, v_2 are strings over O describing multisets of objects. These are *object evolution rules* associated with cells and depending only on the label and the polarization of the cell. The string v_1 has at least one object.
   b) $a[\,]_i^{e_1} \to [b]_i^{e_2}$ where $i \in \{1, \ldots, L\}$, $e_1, e_2 \in P$ and $a, b \in O$. These are *send-in rules*. An object of the environment is introduced in the membrane possibly modified. The polarization of the cell can also change.
   c) $[a]_i^{e_1} \to b[\,]_i^{e_2}$ where $i \in \{1, \ldots, L\}$, $e_1, e_2 \in P$ and $a, b \in O$. These are *send-out rules*. An object is sent out to the environment possibly modified. The polarization of the cell can also change.
   d) $[a]_i^{e_1}, [\,]_{i+1} \to [\,]_i, [b]_{i+1}^{e_2}$
      $[\,]_i, [a]_{i+1}^{e_1} \to [b]_i^{e_2}, [\,]_{i+1}$
      where $i \in \{1, \ldots, L\}$, $e_1, e_2 \in P$ and $a, b \in O$. These are *communication rules*. An object a is sent, possibly modified, to a contiguous cell.

Rules are applied according to the following principles:

- Rules are used as usual in the framework of membrane computing, that is, in a maximal parallel way. In one step, each object in a cell can only be used for one rule (non deterministically chosen when there are several possibilities), but any object which can evolve by a rule of any form must do it. with a restriction, only one change of polarization can affect to a membrane.
- All the elements which are not involved in any of the operations to be applied remain unchanged.
- Several rules can be applied to different objects in the same cell simultaneously.
- If any rule of type (a) are used at the same time that one of type (b), (c) or (d), all rules are applied, but we will consider that the object evolution rules (a) are performed *before* the other one. This consideration is useful because the rule that sends an object across a membrane can also changes its polarization.

4 Modeling BD

In this section we will consider a BD system with L columns and we will provide a deposition P system which simulates its dynamics. Let us consider the deposition P systems of degree L, $\Pi = (O, \mu, env, v_1, \ldots, v_L, v_{env}, P, R)$ where:

- $O = \{p, c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_{n3}, c_{n4}, \alpha, x, y, z\}$
- $v_i = \emptyset$, for all $i \in \{1, \ldots, L\}$
- $v_{env} = p$

Let us consider the following sets of rules, where the index $i \in \{1, \ldots, L\}$. As remarked before, we will identify the indices $L+1$ and 1 and the indices 0 and L; and, if a cell has polarization 0, we will omit the symbol 0.

Set (A) – **Deposition rules:**

$$R^i_* \equiv p\,[\,]_i \rightarrow [c_0]^+_i$$

In the BD model, a particle is deposited on the top of a column randomly chosen. We simulate this process by these rules. A particle p in the environment is sent to one of the cells. This particle activates the cell (the polarization of the cells turns on positive) and goes into the cell as the object c_0.

Set (B) – **Rules for cells with positive polarization:**

$$R^i_1 \equiv [c_0 \rightarrow c_1]^+_i \qquad R^i_4 \equiv [\,]_i, [c_2]^+_{i+1} \rightarrow [c_{n3}]^-_i, [\,]_{i+1}$$
$$R^i_2 \equiv [c_1 \rightarrow c_2]^+_i \qquad R^i_5 \equiv [\,]_i, [z]^+_{i+1} \rightarrow [y]_i, [\,]_{i+1}$$
$$R^i_3 \equiv [y \rightarrow z\,\alpha]^+_i$$

The sets R^i_1, R^i_2 and R^i_3 are object evolution rules and R^i_4 and R^i_5 are communication rules. Note that the counter c_k sent into a cell i, by the particle p gives two waiting step before being sent to the cell $i-1$ transformed into c_{n3} (by the rule R^{i-1}_4). These waiting steps check the occurrence of objects y inside the cell. If any y occur, each of then evolves to $z\,\alpha$ at the same time in which c_0 evolves to c_1 and in the following step each z is sent to the cell $i-1$ transformed into y and the polarization of the cell i changes. If this happens, the rule R_4^{i-1} is not triggered because the cell containing c_2 has not positive charge.

Set (C) – **Rules for cells with negative polarization:**

$$\begin{array}{ll} R_6^i \equiv [c_3 \to c_4]_i^- & R_{10}^i \equiv [c_{n3} \to c_{n4}]_i^- \\ R_7^i \equiv [c_4 \to c_5]_i^- & R_{11}^i \equiv [c_{n4} \to x\, y\, c_5]_i^- \\ R_8^i \equiv [c_5 \to c_6]_i^- & R_{12}^i \equiv [x]_i^-, [\,]_{i+1} \to [\,]_i, [z]_{i+1} \\ R_9^i \equiv [c_6]_i^- \to p\, [\,]_i & \end{array}$$

The sets R_6^i, R_7^i, R_8^i, R_{10}^i and R_{11}^i are object evolution rules, R_{12}^i are communication rules and R_9^i are send-out rules. The counter c_k goes on with the objects c_3 and c_4 or with c_{n3} and c_{n4}. In both cases, the counter reaches c_5 and c_6. The object c_6 sends to the environment an object p which will go into a cell in the next step according to the set of rules R_*^i.

Set (D) – **Rules for cells with polarization zero:**

$$\begin{array}{ll} R_{13}^i \equiv [c_{n4} \to c_5]_i & R_{16}^i \equiv [z \to x\, \alpha]_i \\ R_{14}^i \equiv [c_5 \to c_6]_i & R_{17}^i \equiv [x\, y \to \lambda]_i \\ R_{15}^i \equiv [c_6]_i \to p\, [\,]_i & R_{18}^i \equiv [\,]_i, [c_2]_{i+1} \to [c_3]_i^-, [\,]_{i+1} \end{array}$$

The sets R_{13}^i, R_{14}^i, R_{16}^i, and R_{17}^i are object evolution rules, R_{18}^i are communication rules and R_{15}^i are send-out rules. As in the set of rule (B), the counter c_k goes on till it reaches c_6. The object c_6 sends to the environment an object p which will go into a cell in the next step according to the set of rules R_*^i. Notice that rules of R_{17}^i are cooperative rules. Each pair of objects $\langle x, y \rangle$ which occur in a cell disappears in the next step.

4.1 Informal description of the computation

For a better understanding of the computation, let us remark that the configurations at time $8t$ with $t \in \mathbb{N}$ represent the state of a BD system after the deposition of the t-th particle. Below we will formalize this idea, but before giving a description of the computation, we provide the intuitive meaning of some of the symbols of the alphabet at time $8t$:

- p represents the particle that arrives to the substrate. When it is deposited, it disappears from the environment, then the information encoded in the cells change. When the computation inside the cells finishes, a new particle is sent to the environment and the process starts again. In this way only in the steps $8t$, there exists a particle p in the environment.
- The multiplicity of α in the cell i represent the height of the column i in the BD model.

Time	**Rules**	**Env.**	**Configuration**
T_0		p	$-[\ ]_1 - [\ ]_2 - [\ ]_3 - [\ ]_4 -$
T_1	R^3_*		$-[\ ]_1 - [\ ]_2 - [\,c_0\,]^+_3 - [\ ]_4 -$
T_2	R^3_1		$-[\ ]_1 - [\ ]_2 - [\,c_1\,]^+_3 - [\ ]_4 -$
T_3	R^3_2		$-[\ ]_1 - [\ ]_2 - [\,c_2\,]^+_3 - [\ ]_4 -$
T_4	R^2_4		$-[\ ]_1 - [\,c_{n3}\,]^-_2 - [\ ]_3 - [\ ]_4 -$
T_5	R^2_{10}		$-[\ ]_1 - [\,c_{n4}\,]^-_2 - [\ ]_3 - [\ ]_4 -$
T_6	R^2_{11}		$-[\ ]_1 - [\,x\,y\,c_5\,]^-_2 - [\ ]_3 - [\ ]_4 -$
T_7	R^2_8, R^2_{12}		$-[\ ]_1 - [\,y\,c_6\,]_2 - [\,z\,]_3 - [\ ]_4 -$
T_8	R^2_{15}, R^3_{16}	p	$-[\ ]_1 - [\,y\,]_2 - [\,x\,\alpha\,]_3 - [\ ]_4 -$

Fig. 2. Table with the first steps

Finally, the remaining objects inside the cells at time $8t$ are of type x and y. The objects x or y inside the cell i represent the difference of height between the cell i and the cell $i+1$.

- The multiplicity of x in the cell i represent how many units is the column i higher than column $i+1$ in the BD model.
- The multiplicity of y in the cell i represent how many units is the column i lower than column $i+1$ in the BD model.

From the previous description we have that at time $8t$, we can find inside a cell objects x, y or none of them, but we will never find both simultaneously.

Tables 2 and 3 show an example of evolution of a simple BD system with four columns where four particles have been deposited sequentially on the columns 3, 2, 2 and 1. Notice that the configurations at times $8t$ with $t \in \{0, \dots, 4\}$ represent the *surface* of the BD model after the fall of the t-th particle (Fig. 4).

We finish this section by formally showing that this deposition P system of degree L simulates the ballistic deposition on a substrate with L columns. In this way we need the definition of *representative configuration*. The idea behind the

Time	Rules	Env.	Configuration
T_9	R^2_*		$-\ [\ ]_1 - [y\,c_0]_2^{+} - [x\,\alpha]_3 - [\ ]_4\ -$
T_{10}	R^1_2, R^2_3		$-\ [\ ]_1 - [z\,\alpha\,c_1]_2^{+} - [x\,\alpha]_3 - [\ ]_4\ -$
T_{11}	R^2_2, R^1_5		$-\ [y]_1 - [\alpha\,c_2]_2 - [x\,\alpha]_3 - [\ ]_4\ -$
T_{12}	R^1_{18}		$-\ [y\,c_3]_1^{-} - [\alpha]_2 - [x\,\alpha]_3 - [\ ]_4\ -$
T_{13}	R^1_6		$-\ [y\,c_4]_1^{-} - [\alpha]_2 - [x\,\alpha]_3 - [\ ]_4\ -$
T_{14}	R^1_7		$-\ [y\,c_5]_1^{-} - [\alpha]_2 - [x\,\alpha]_3 - [\ ]_4\ -$
T_{15}	R^1_8		$-\ [y\,c_6]_1^{-} - [\alpha]_2 - [x\,\alpha]_3 - [\ ]_4\ -$
T_{16}	R^1_9	p	$-\ [y]_1 - [\alpha]_2 - [x\,\alpha]_3 - [\ ]_4\ -$
...	...	...	$-\ [\dots]_1 - [\dots]_2 - [\dots]_3 - [\dots]_4\ -$
T_{24}	...	p	$-\ [y^2]_1 - [x\,\alpha^2]_2 - [\alpha\,x]_3 - [\ ]_4\ -$
...	...	...	$-\ [\dots]_1 - [\dots]_2 - [\dots]_3 - [\dots]_4\ -$
T_{32}	...	p	$-\ [\alpha^2]_1 - [x\,\alpha^2]_2 - [x\,\alpha]_3 - [y^2]_4\ -$

Fig. 3. Table with the first steps (Cont.)

definition is quite intuitive. Along the computation, some of the configurations have no meaning with respect to the deposition process, there are merely auxiliary steps of the computation. Only some of the configuration represent states of the aggregate in the deposition process. Such configurations will be called *representative configurations*

We will denote by C_t the configuration of the P systems at time t, by $C_t(i)$ the multiset of objects at the cell labelled by i at time t and by $|C_t(i)|_a$ the multiplicity of the object a in $C_t(i)$.

Definition 1. *Let C_t be a configuration of a deposition P systems of degree L at time t. We will say that C_t is representative if for all $i \in \{1, \dots, L\}$,*

- *Only objects α, x and y occur inside the cells and they polarization* 0

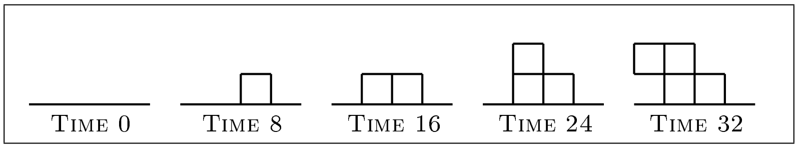

Fig. 4. Example

- $C_t(env) = \{p\}$
- *If* $|C_t(i)|_\alpha \geq |C_t(i+1)|_\alpha$ *then*
  - $|C_t(i)|_x = |C_t(i)|_\alpha - |C_t(i+1)|_\alpha$
  - $|C_t(i)|_y = 0$
- *If* $|C_t(i)|_\alpha < |C_t(i+1)|_\alpha$ *then*
  - $|C_t(i)|_y = |C_t(i+1)|_\alpha - |C_t(i)|_\alpha$
  - $|C_t(i)|_x = 0$

Finally, the next theorem claims that the sequence of configurations at times $0, 8, 16, \ldots, 8t, \ldots$ represent the states of an aggregate with a ballistic deposition process.

Theorem 1. *For all* $t \in \mathbb{N}$, C_{8t} *is a representative configuration and, if* $i \in \{1, \ldots, L\}$ *is the chosen cell for depositing a new particle, then*

- $|C_{8t+8}(i)|_\alpha = max\{|C_{8t}(i-1)|_\alpha, |C_{8t}(i)|_\alpha + 1, |C_{8t}(i+1)|_\alpha\}$
- *For all* $j \in \{1, \ldots, L\}$, $i \neq j$, $|C_{8t+8}(j)|_\alpha = |C_{8t}(j)|_\alpha$

Proof. See Appendix

5 Conclusions

Understanding how Nature works involves experimental observation and theoretical modeling. This paper is a contribution to the theoretical modeling of a particular case of one of the most interesting process in Physics: the dynamical evolution of the frontier between two different media. In this paper, the chosen model has been Ballistic Deposition, but many other deposition processes from Physics, Chemistry and Biology can be also modeled by using similar techniques.

On the other hand, Membrane Computing techniques had been used for studying problems from many different areas, since Linguistics or Complexity Theory to Computer Graphics or Cancer Modeling[2], but this is the first time that P systems are used to model deposition processes.

This paper can be extended in several ways. One of them is to extend the study to more dimensions, i.e., to consider the particles as 3D solids falling down

[2] See [4] for details.

onto a 2D surface. Other possible research line is to develop computer software which simulates Ballistic Depositions according with the Membrane Computing techniques presented in this paper and of course, a final research line is to follow this study by modeling other deposition models.

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6 Appendix

Proof of the theorem 1.

Proof. The proof is by induction on $t \in \mathbb{N}$. The key idea of the proof is to notice that the P system is deterministic with the only exception of the set of rules $R^* \equiv p\,[\,]_i \rightarrow [c_0]_i^+$ for $i \in \{1, \ldots, L\}$. This set of rules represent the non-deterministic choice of a cell in order to deposit a new particle.

$\boxed{\text{t=0}}$

In the initial configuration C_0 all cells are empty and have polarisation 0; also, $C_0(env) = \{p\}$, then it is a representative configuration.
Let us suppose that i is the chosen cell in the non-deterministic step. The first configurations are

$C_0(i) = \{\ \} \xrightarrow{R_*^i} C_1(i) = \{c_0\}^+ \xrightarrow{R_1^i} C_2(i) = \{c_1\}^+ \xrightarrow{R_2^i} C_3(i) = \{c_2\}^+$

$C_k(j) = \emptyset$, for all $j \in \{1, \ldots, i-1, i+1, \ldots, L, env\}$ and $k \in \{1, 2, 3\}$ as no rules affect to them.
As the cell i has positive electrical charge in C_3, then we apply the rule $[\,]_{i-1}, [c_2]_i^+ \to [c_{n3}]_{i-1}^-, [\,]_i$ obtaining

$C_4(i-1) = \{c_{n3}\}^-$

$C_4(j) = \emptyset$ for all $j \in \{1, \ldots, i-2, i, \ldots, L, env\}$

Hence

$C_4(i-1) = \{c_{n3}\}^- \xrightarrow{R_{10}^{i-1}} C_5(i-1) = \{c_{n4}\}^- \xrightarrow{R_{11}^{i-1}} C_6(i-1) = \{x\, y\, c_{n5}\}$

$C_k(j) = \emptyset$ for all $j \in \{1, \ldots, i-2, i, \ldots, L, env\}$ and $k \in \{5, 6\}$ as no rules affect to them.
At this step rules $[c_5 \to c_6]_{i-1}^-$ and $[x]_{i-1}^-, [\,] \to [\,]_{i-1}[z]_i$ are applied simultaneously

$C_7(i-1) = \{c_6\, y\}$

$C_7(i) = \{z\}$

$C_7(j) = \emptyset$ for all $j \in \{1, \ldots, i-2, i+1, \ldots, L, env\}$

Finally, rules $[c_6]_{i-1} \to p\,[\,]_{i-1}$ and $[z \to x\,\alpha]_i$ are applied

$C_8(i-1) = \{y\}$

$C_8(i) = \{x\,\alpha\}$

$C_8(env) = \{p\}$

$C_8(j) = \emptyset$ for all $j \in \{1, \ldots, i-2, i+1, \ldots, L\}$

In configuration C_8, all the cells has polarisation 0 and only objects x, y and α occur inside them, $C_8(env) = \{p\}$ and

$|C_8(i-1)|_\alpha = 0 < |C_8(i)|_\alpha = 1$

$|C_8(i-1)|_y = 1 = |C_8(i)|_\alpha - |C_8(i-1)|_\alpha$

$|C_8(i-1)|_x = 0$

$|C_8(i)|_\alpha = 1 \geq |C_8(i+1)|_\alpha = 0$

$|C_8(i)|_x = 1 = |C_8(i)|_\alpha - |C_8(i+1)|_\alpha$

$|C_8(i)|_y = 0$

$C_8(j) = \emptyset$ for all $j \in \{1, \ldots, i-2, i+1, \ldots, L\}$

Hence, C_8 is a representative configuration. We also have that

$|C_8(j)|_\alpha = |C_0(j)|_\alpha$ for $j \in \{1, \ldots, i-1, i+1, \ldots, L\}$

$|C_8(i)|_\alpha = max\{|C_0(i-1)|_\alpha, |C_0(i)|_\alpha + 1, |C_0(i+1)|_\alpha\} = 1$

$\boxed{\text{t} \to \text{t+1}}$

Let us suppose that C_{8t} is a representative configuration and i is the cell chosen non-deterministically. We will prove that C_{8t+8} is also a representative configuration and

$$|C_{8t+8}(i)|_\alpha = max\{|C_{8t}(i-1)|_\alpha, |C_{8t}(i)|_\alpha + 1, |C_{8t}(i+1)|_\alpha\}$$

For all $j \in \{1, \ldots, L\}, i \neq j, |C_{8t+8}(j)|_\alpha = |C_{8t}(j)|_\alpha$

We will consider five possible cases depending on the relation among $|C_{8t}(i-1)|_\alpha$, $|C_{8t}(i)|_\alpha$ and $|C_{8t}(i+1)|_\alpha$.

Case 1: $|C_{8t}(i-1)|_\alpha > |C_{8t}(i)|_\alpha \geq |C_{8t}(i+1)|_\alpha$
Case 2: $|C_{8t}(i-1)|_\alpha > |C_{8t}(i+1)|_\alpha > |C_{8t}(i)|_\alpha$
Case 3: $|C_{8t}(i+1)|_\alpha \geq |C_{8t}(i-1)|_\alpha > |C_{8t}(i)|_\alpha$
Case 4: $|C_{8t}(i)|_\alpha \geq |C_{8t}(i-1)|_\alpha$ and $|C_{8t}(i)|_\alpha \geq |C_{8t}(i+1)|_\alpha$
Case 5: $|C_{8t}(i+1)|_\alpha > |C_{8t}(i)|_\alpha \geq |C_{8t}(i-1)|_\alpha$

The proof is made by inspection of these cases.

Case 1: Let us suppose that

$$|C_{8t}(i-1)|_\alpha > |C_{8t}(i)|_\alpha \geq |C_{8t}(i+1)|_\alpha$$

In this case $|C_{8t}(i-1)|_x = |C_{8t}(i-1)|_\alpha - |C_{8t}(i)|_\alpha > 0$, $|C_{8t}(i)|_x = |C_{8t}(i)|_\alpha - |C_{8t}(i+1)|_\alpha \geq 0$ and $|C_{8t}(i-1)|_y = |C_{8t}(i)|_y = 0$.
Also, $max\{|C_{8t}(i-1)|_\alpha, |C_{8t}(i)|_\alpha + 1, |C_{8t}(i+1)|_\alpha\} = |C_{8t}(i-1)|_\alpha$. We will prove that C_{8t+8} is a representative configuration, $|C_{8t+8}(i)|_\alpha = |C_{8t}(i-1)|_\alpha$ and $|C_{8t+8}(j)|_\alpha = |C_{8t}(j)|_\alpha$ for $j \in \{1, \ldots, i-1, i+1, \ldots, L\}$
As no y are present in cell i,

$C_{8t}(i) \xrightarrow{R^i_*} C_{8t+1}(i) = C_{8t}(i) \cup \{c_0\}^+ \xrightarrow{R^i_1} C_{8t+2}(i) = C_{8t}(i) \cup \{c_1\}^+ \xrightarrow{R^i_2} C_{8t+3}(i) = C_{8t}(i) \cup \{c_2\}^+$

$C_{8t+k}(j) = C_{8t}(j)$, for all $j \in \{1, \ldots, i-1, i+1, \ldots, L\}$ as no rules affect to them and $C_{8t+k}(env) = \emptyset$ for $k \in \{1, 2, 3\}$.
As cell i has positive electrical charge in C_{8t+3}, then we apply the rule $[\,]_{i-1}, [c_2]^+_i \to [c_{n3}]^-_{i-1}, [\,]_i$ obtaining

$C_{8t+4}(i-1) = C_{8t}(i-1) \cup \{c_{n3}\}^-$

$C_{8t+4}(j) = C_{8t}(j)$ for all $j \in \{1, \ldots, i-2, i, \ldots, L\}$ and $C_{8t+4}(env) = \emptyset$

At this step rules $[c_{n3} \to c_{n4}]^-_{i-1}$ and $[x]^-_{i-1}, [\,] \to [\,]_{i-1}[z]_i$ are applied simultaneously and therefore all the copies of x in cell $i-1$ are sent into cell i transformed into copies of z. Hence

$C_{8t+5}(i-1) = \{\alpha^{|C_{8t}(i-1)|_\alpha}\, c_{n4}\}$

$C_{8t+5}(i) = C_{8t}(i) \cup \{z^{|C_{8t}(i-1)|_x}\}$

$C_{8t+5}(j) = C_{8t}(j)$ for all $j \in \{1, \ldots, i-2, i+1, \ldots, L\}$ and $C_{8t+5}(env) = \emptyset$

Now, rules $[c_{n4} \to c_5]_{i-1}$ and $[z \to x\,\alpha]_i$ can be applied, then

$C_{8t+6}(i-1) = \{\alpha^{|C_{8t}(i-1)|_\alpha}\, c_5\}$

$C_{8t+6}(i) = C_{8t}(i) \cup \{x^{|C_{8t}(i-1)|_x}\, \alpha^{|C_{8t}(i-1)|_x}\} =$
$= \{x^{|C_{8t}(i)|_x + |C_{8t}(i-1)|_x}\, \alpha^{|C_{8t}(i)|_\alpha + |C_{8t}(i-1)|_x}\}$ as $|C_{8t}(i-1)|_y = 0$

$C_{8t+6}(j) = C_{8t}(j)$ for all $j \in \{1, \ldots, i-2, i+1, \ldots, L\}$ and $C_{8t+6}(env) = \emptyset$

After that,

$C_{8t+6}(i-1) = \{\alpha^{|C_{8t}(i-1)|_\alpha}\, c_5\} \xrightarrow{R^{i-1}_8} C_{8t+7}(i-1) = \{\alpha^{|C_{8t}(i-1)|_\alpha}\, c_6\}$

$C_{8t+7}(i) = C_{8t+6}(i)$
$C_{8t+7}(j) = C_{8t}(j)$ for all $j \in \{1, \ldots, i-2, i+1, \ldots, L\}$ and $C_{8t+7}(env) = \emptyset$
Next, the rule $[c_6]_{i-1} \rightarrow p\,[\,]_{i-1}$ is applied,
$C_{8t+8}(i-1) = \{\alpha^{|C_{8t}(i-1)|_\alpha}\}$
$C_{8t+8}(i) = C_{8t+6}(i) = \{x^{|C_{8t}(i)|_x + |C_{8t}(i-1)|_x}\ \alpha^{|C_{8t}(i)|_\alpha + |C_{8t}(i-1)|_x}\}$
$C_{8t+8}(j) = C_{8t}(j)$ for all $j \in \{1, \ldots, i-1, i+1, \ldots, L\}$ and $C_{8t+8}(env) = \{p\}$.
Note that $|C_{8t+8}(i)|_\alpha = |C_{8t}(i)|_\alpha + |C_{8t}(i-1)|_x$ and by hypothesis $|C_{8t}(i-1)|_x = |C_{8t}(i-1)|_\alpha - |C_{8t}(i)|_\alpha$ so $|C_{8t+8}(i)|_\alpha = |C_{8t}(i-1)|_\alpha = |C_{8t+8}(i-1)|_\alpha$, and that there is no x or y in $C_{8t+8}(i-1)$.
Finally as in this case $|C_{8t+8}(i)|_\alpha = |C_{8t}(i-1)|_\alpha > |C_{8t}(i)|_\alpha$ and $|C_{8t}(i)|_\alpha \geq |C_{8t}(i+1)|_\alpha = |C_{8t+8}(i+1)|_\alpha$, then

$$|C_{8t+8}(i)|_\alpha > |C_{8t+8}(i+1)|_\alpha$$

and

$$\begin{aligned} |C_{8t+8}(i)|_x &\overset{(1)}{=} |C_{8t}(i)|_x + |C_{8t}(i-1)|_x \\ &\overset{(2)}{=} (|C_{8t}(i)|_\alpha - |C_{8t}(i+1)|_\alpha) + |C_{8t}(i-1)|_x \\ &\overset{(3)}{=} (|C_{8t}(i)|_\alpha + |C_{8t}(i-1)|_x) - |C_{8t+8}(i+1)|_\alpha \\ &\overset{(4)}{=} |C_{8t+8}(i)|_\alpha - |C_{8t+8}(i+1)|_\alpha \end{aligned}$$

The equality (1) holds by the explicit description obtained for $C_{8t+8}(i)$. The equality (2) holds by hypothesis of induction. The third equality holds because for all $j \in \{1, \ldots, L\}$, $i \neq j$, $|C_{8t+8}(j)|_\alpha = |C_{8t}(j)|_\alpha$, in particular for $j = i+1$ and the last equality holds by the explicit description obtained for $C_{8t+8}(i)$.
In order to finish the proof that C_{8t+8} is a representative configuration we need to see that $|C_{8t+8}(i)|_y = 0$ but this holds by the explicit description obtained for $C_{8t+8}(i)$.

Case 2: Let us suppose that

$$|C_{8t}(i-1)|_\alpha > |C_{8t}(i+1)|_\alpha > |C_{8t}(i)|_\alpha$$

In this case $|C_{8t}(i-1)|_x = |C_{8t}(i-1)|_\alpha - |C_{8t}(i)|_\alpha > 0$, $|C_{8t}(i)|_x = |C_{8t}(i-1)|_y = 0$ and $|C_{8t}(i)|_y = |C_{8t}(i+1)|_\alpha - |C_{8t}(i)|_\alpha > 0$.
Also, $max\{|C_{8t}(i-1)|_\alpha, |C_{8t}(i)|_\alpha + 1, |C_{8t}(i+1)|_\alpha\} = |C_{8t}(i-1)|_\alpha$. We will prove that C_{8t+8} is a representative configuration, $|C_{8t+8}(i)|_\alpha = |C_{8t}(i-1)|_\alpha$ and $|C_{8t+8}(j)|_\alpha = |C_{8t}(j)|_\alpha$ for $j \in \{1, \ldots, i-1, i+1, \ldots, L\}$
As y is present in cell i,
$C_{8t}(i) \xrightarrow{R^i_*} C_{8t+1}(i) = C_{8t}(i) \cup \{c_0\}^+ \xrightarrow{R^i_1, R^i_3} C_{8t+2}(i) = \{\alpha^{|C_{8t}(i)|_\alpha}\} \cup \{z^{|C_{8t}(i)|_y}\ \alpha^{|C_{8t}(i)|_y}\ c_1\}^+$
$C_{8t+k}(j) = C_{8t}(j)$, for all $j \in \{1, \ldots, i-1, i+1, \ldots, L\}$ as no rules affect to them and $C_{8t+k}(env) = \emptyset$ for $k \in \{1, 2\}$.
As cell i has positive electrical charge in C_{8t+2}, then we apply the rules $[\,]_{i-1}, [z]^+_i \rightarrow [y]_{i-1}, [\,]_i$ and $[c_1 \rightarrow c_2]^+_i$ obtaining
$C_{8t+3}(i-1) = C_{8t}(i-1) \cup \{y^{|C_{8t}(i)|_y}\}$

$C_{8t+3}(i) = \{\alpha^{|C_{8t}(i)|_\alpha + |C_{8t}(i)|_y}\, c_2\}$
$C_{8t+3}(j) = C_{8t}(j)$, for all $j \in \{1, \ldots, i-2, i+1, \ldots, L\}$ and $C_{8t+3}(env) = \emptyset$.
In this situation $|C_{8t+3}(i-1)|_x = |C_{8t}(i-1)|_x > 0$ and $|C_{8t+3}(i-1)|_y = |C_{8t}(i)|_y > 0$ so the rule $[x\,y \to \lambda]_{i-1}$ can be applied. In order to compute the number of copies of x and y that remain in cell $i-1$ after the application of this rule we consider that

$$\begin{aligned} |C_{8t}(i-1)|_x &\overset{(1)}{=} |C_{8t}(i-1)|_\alpha - |C_{8t}(i)|_\alpha \\ &\overset{(2)}{>} |C_{8t}(i+1)|_\alpha - |C_{8t}(i)|_\alpha \\ &\overset{(3)}{=} |C_{8t}(i)|_y \end{aligned}$$

The equality (1) holds because C_{8t} is a representative configuration and in this case $|C_{8t}(i-1)|_\alpha > |C_{8t}(i)|_\alpha$. The inequality (2) holds because in this case $|C_{8t}(i-1)|_\alpha > |C_{8t}(i+1)|_\alpha$ and finally, the last equality holds by the definition of representative configuration and $|C_{8t}(i+1)|_\alpha > |C_{8t}(i)|_\alpha$.
The rule $[\,]_{i-1}, [c_2]_i^+ \to [c_3]_{i-1}^-, [\,]_i$ can also be applied. Therefore,
$C_{8t+4}(i-1) = \{x^{|C_{8t}(i-1)|_x - |C_{8t}(i)|_y}\, \alpha^{|C_{8t}(i-1)|_\alpha}\, c_3\}^-$
$C_{8t+4}(i) = \{\alpha^{|C_{8t}(i)|_\alpha + |C_{8t}(i)|_y}\}$
$C_{8t+4}(j) = C_{8t}(j)$, for all $j \in \{1, \ldots, i-2, i+1, \ldots, L\}$ and $C_{8t+4}(env) = \emptyset$.
Since polarisation of cell $i-1$ is now negative, rules $[x]_{i-1}^-, [\,]_i \to [\,]_{i-1}, [z]_i$ and $[c_3 \to c_4]_{i-1}^-$ can be applied and all the copies of x from cell $i-1$ are sent into the cell i transformed into copies of z. Hence
$C_{8t+5}(i-1) = \{\alpha^{|C_{8t}(i-1)|_\alpha}\, c_4\}$
$C_{8t+5}(i) = \{z^{|C_{8t}(i-1)|_x - |C_{8t}(i)|_y}\, \alpha^{|C_{8t}(i)|_\alpha + |C_{8t}(i)|_y}\}$
$C_{8t+5}(j) = C_{8t}(j)$, for all $j \in \{1, \ldots, i-2, i+1, \ldots, L\}$ and $C_{8t+5}(env) = \emptyset$.
After that, rules $[c_{n4} \to c_5]_i$ and $[z \to x\,\alpha]_i$ can be applied, then
$C_{8t+6}(i-1) = \{\alpha^{|C_{8t}(i-1)|_\alpha}\, c_5\}$

$$\begin{aligned} C_{8t+6}(i) &= \{x^{|C_{8t}(i-1)|_x - |C_{8t}(i)|_y}\, \alpha^{|C_{8t}(i-1)|_x - |C_{8t}(i)|_y + |C_{8t}(i)|_\alpha + |C_{8t}(i)|_y}\} = \\ &= \{x^{|C_{8t}(i-1)|_x - |C_{8t}(i)|_y}\, \alpha^{|C_{8t}(i-1)|_x + |C_{8t}(i)|_\alpha}\} \end{aligned}$$

$C_{8t+6}(j) = C_{8t}(j)$, for all $j \in \{1, \ldots, i-2, i+1, \ldots, L\}$ and $C_{8t+6}(env) = \emptyset$.
In next step only the rule $[c_5 \to c_6]_{i-1}$ can be applied and we obtain
$C_{8t+7}(i-1) = \{\alpha^{|C_{8t}(i-1)|_\alpha}\, c_6\}$
$C_{8t+7}(i) = C_{8t+6}(i)$
$C_{8t+7}(j) = C_{8t}(j)$, for all $j \in \{1, \ldots, i-2, i+1, \ldots, L\}$ and $C_{8t+7}(env) = \emptyset$.
Next, the rule $[c_6]_{i-1} \to p\,[\,]_{i-1}$ is applied
$C_{8t+8}(i-1) = \{\alpha^{|C_{8t}(i-1)|_\alpha}\}$
$C_{8t+8}(i) = C_{8t+6}(i) = \{x^{|C_{8t}(i-1)|_x - |C_{8t}(i)|_y}\, \alpha^{|C_{8t}(i-1)|_x + |C_{8t}(i)|_\alpha}\}$
$C_{8t+8}(j) = C_{8t}(j)$, for all $j \in \{1, \ldots, i-2, i+1, \ldots, L\}$ and $C_{8t+8}(env) = \{p\}$.
Note that $|C_{8t+8}(i)|_\alpha = |C_{8t}(i-1)|_x + |C_{8t}(i)|_\alpha$ and by hypothesis $|C_{8t}(i-1)|_x = |C_{8t}(i-1)|_\alpha - |C_{8t}(i)|_\alpha$ so $|C_{8t+8}(i)|_\alpha = |C_{8t}(i-1)|_\alpha = |C_{8t+8}(i-1)|_\alpha$, and that there is no x or y in $C_{8t+8}(i-1)$.
Finally as in this case $|C_{8t}(i-1)|_\alpha > |C_{8t}(i+1)|_\alpha = |C_{8t+8}(i+1)|_\alpha$ then

$$|C_{8t+8}(i)|_\alpha > |C_{8t+8}(i+1)|_\alpha$$

and

$$\begin{aligned} |C_{8t+8}(i)|_x &\overset{(1)}{=} |C_{8t}(i-1)|_x - |C_{8t}(i)|_y = \\ &\overset{(2)}{=} (|C_{8t}(i-1)|_\alpha - |C_{8t}(i)|_\alpha) - (|C_{8t}(i+1)|_\alpha - |C_{8t}(i)|_\alpha) = \\ &\overset{(3)}{=} |C_{8t}(i-1)|_\alpha - |C_{8t}(i+1)|_\alpha \\ &\overset{(4)}{=} |C_{8t+8}(i)|_\alpha - |C_{8t+8}(i+1)|_\alpha \end{aligned}$$

The equality (1) holds by the explicit description obtained for $C_{8t+8}(i)$. The equality (2) holds by hypothesis of induction. The third equality by arithmetic and the last equality holds by the above reasoning about $|C_{8t+8}(i)|_\alpha$ and because for all $j \in \{1,\ldots,L\}$, $i \neq j$, $|C_{8t+8}(j)|_\alpha = |C_{8t}(j)|_\alpha$, in particular for $j = i+1$.
In order to finish the proof that C_{8t+8} is a representative configuration we need to see that $|C_{8t+8}(i)|_y = 0$ but this holds by the explicit description obtained for $C_{8t+8}(i)$.

Case 3: Let us suppose that

$$|C_{8t}(i+1)|_\alpha \geq |C_{8t}(i-1)|_\alpha > |C_{8t}(i)|_\alpha$$

In this case $|C_{8t}(i-1)|_x = |C_{8t}(i-1)|_\alpha - |C_{8t}(i)|_\alpha > 0$, $|C_{8t}(i)|_x = |C_{8t}(i-1)|_y = 0$ and $|C_{8t}(i)|_y = |C_{8t}(i+1)|_\alpha - |C_{8t}(i)|_\alpha > 0$.
Also, $max\{|C_{8t}(i-1)|_\alpha, |C_{8t}(i)|_\alpha + 1, |C_{8t}(i+1)|_\alpha\} = |C_{8t}(i+1)|_\alpha$. We will prove that C_{8t+8} is a representative configuration, $|C_{8t+8}(i)|_\alpha = |C_{8t}(i+1)|_\alpha$ and $|C_{8t+8}(j)|_\alpha = |C_{8t}(j)|_\alpha$ for $j \in \{1,\ldots,i-1,i+1,\ldots,L\}$
As y is present in cell i,

$C_{8t}(i) \xrightarrow{R^i_*} C_{8t+1}(i) = C_{8t}(i) \cup \{c_0\}^+ \xrightarrow{R^i_1, R^i_3} C_{8t+2}(i) = \{\alpha^{|C_{8t}(i)|_\alpha}\} \cup \{z^{|C_{8t}(i)|_y}\ \alpha^{|C_{8t}(i)|_y}\ c_1\}^+$

$C_{8t+k}(j) = C_{8t}(j)$, for all $j \in \{1,\ldots,i-1,i+1,\ldots,L\}$ as no rules affect to them and $C_{8t+k}(env) = \emptyset$ for $k \in \{1,2\}$.
As cell i has positive electrical charge in C_{8t+2}, then we apply the rules $[\,]_{i-1}, [z]_i^+ \to [y]_{i-1}, [\,]_i$ and $[c_1 \to c_2]_i^+$ obtaining

$C_{8t+3}(i-1) = C_{8t}(i-1) \cup \{y^{|C_{8t}(i)|_y}\}$

$C_{8t+3}(i) = \{\alpha^{|C_{8t}(i)|_\alpha + |C_{8t}(i)|_y}\ c_2\}$

$C_{8t+3}(j) = C_{8t}(j)$, for all $j \in \{1,\ldots,i-2,i+1,\ldots,L\}$ and $C_{8t+3}(env) = \emptyset$.

In this situation $|C_{8t+3}(i-1)|_x = |C_{8t}(i-1)|_x > 0$ and $|C_{8t+3}(i-1)|_y = |C_{8t}(i)|_y > 0$ so the rule $[x\,y \to \lambda]_{i-1}$ can be applied. In order to compute the number of copies of x and y that remain in cell $i-1$ after the application of this rule we consider that

$$\begin{aligned} |C_{8t}(i-1)|_x &\overset{(1)}{=} |C_{8t}(i-1)|_\alpha - |C_{8t}(i)|_\alpha \\ &\overset{(2)}{\leq} |C_{8t}(i+1)|_\alpha - |C_{8t}(i)|_\alpha \\ &\overset{(3)}{=} |C_{8t}(i)|_y \end{aligned}$$

The equality (1) holds because C_{8t} is a representative configuration and in this case $|C_{8t}(i-1)|_\alpha > |C_{8t}(i)|_\alpha$. The inequality (2) holds because in this case $|C_{8t}(i+1)|_\alpha \geq |C_{8t}(i-1)|_\alpha$ and finally, the last equality holds by the definition of representative configuration and $|C_{8t}(i+1)|_\alpha > |C_{8t}(i)|_\alpha$.
The rule $[\,]_{i-1}, [c_2]_i^+ \to [c_3]_{i-1}^-, [\,]_i$ can also be applied. Therefore,

$C_{8t+4}(i-1) = \{y^{|C_{8t}(i)|_y - |C_{8t}(i-1)|_x}\ \alpha^{|C_{8t}(i-1)|_\alpha}\ c_3\}^-$

$C_{8t+4}(i) = \{\alpha^{|C_{8t}(i)|_\alpha + |C_{8t}(i)|_y}\}$

$C_{8t+4}(j) = C_{8t}(j)$, for all $j \in \{1, \ldots, i-2, i+1, \ldots, L\}$ and $C_{8t+4}(env) = \emptyset$.

As no x are present in cell $i-1$,

$C_{8t+4}(i-1) \xrightarrow{R_6^{i-1}} C_{8t+5}(i-1) = \{y^{|C_{8t}(i)|_y - |C_{8t}(i-1)|_x}\ \alpha^{|C_{8t}(i-1)|_\alpha}\ c_4\}^- \xrightarrow{R_7^{i-1}} C_{8t+6}(i-1) = \{y^{|C_{8t}(i)|_y - |C_{8t}(i-1)|_x}\ \alpha^{|C_{8t}(i-1)|_\alpha}\ c_5\}^- \xrightarrow{R_8^{i-1}} C_{8t+7}(i-1) = \{y^{|C_{8t}(i)|_y - |C_{8t}(i-1)|_x}\ \alpha^{|C_{8t}(i-1)|_\alpha}\ c_6\}^-$

$C_{8t+k}(i) = C_{8t+4}(i)$ and $C_{8t+k}(j) = C_{8t}(j)$ for all $j \in \{1, \ldots, i-2, i+1, \ldots, L\}$ as no rules affect to them and $C_{8t+k}(env) = \emptyset$ for $k \in \{5, 6, 7\}$.

Next, the rule $[c_6]_{i-1}^- \to p\,[\,]_{i-1}$ is applied,

$C_{8t+8}(i-1) = \{y^{|C_{8t}(i)|_y - |C_{8t}(i-1)|_x}\ \alpha^{|C_{8t}(i-1)|_\alpha}\}$

$C_{8t+8}(i) = C_{8t+4}(i)$ and $C_{8t+8}(j) = C_{8t}(j)$ for all $j \in \{1, \ldots, i-2, i+1, \ldots, L\}$ and $C_{8t+8}(env) = \{p\}$.

Note that $|C_{8t+8}(i)|_\alpha = |C_{8t}(i)|_y + |C_{8t}(i)|_\alpha$ and by hypothesis $|C_{8t}(i)|_y = |C_{8t}(i+1)|_\alpha - |C_{8t}(i)|_\alpha$ so $|C_{8t+8}(i)|_\alpha = |C_{8t}(i+1)|_\alpha$, and that there is no x or y in $C_{8t+8}(i)$.
Finally as in this case $|C_{8t+8}(i)|_\alpha = |C_{8t}(i+1)|_\alpha \geq |C_{8t}(i-1)|_\alpha = |C_{8t+8}(i-1)|_\alpha$ then

$$|C_{8t+8}(i)|_\alpha \geq |C_{8t+8}(i-1)|_\alpha$$

and

$$\begin{aligned}
|C_{8t+8}(i-1)|_y &\overset{(1)}{=} |C_{8t}(i)|_y - |C_{8t}(i-1)|_x = \\
&\overset{(2)}{=} (|C_{8t}(i+1)|_\alpha - |C_{8t}(i)|_\alpha) - (|C_{8t}(i-1)|_\alpha - |C_{8t}(i)|_\alpha) = \\
&\overset{(3)}{=} |C_{8t}(i+1)|_\alpha - |C_{8t}(i-1)|_\alpha = \\
&\overset{(4)}{=} |C_{8t}(i+1)|_\alpha - |C_{8t+8}(i)|_\alpha
\end{aligned}$$

The equality (1) holds by the explicit description obtained for $|C_{8t+8}(i-1)|_y$. The equality (2) holds by hypothesis of induction. The third equality by arithmetic and the last by the above reasoning about $|C_{8t+8}(i)|_\alpha$.
In order to finish the proof that C_{8t+8} is a representative configuration we need to see that $|C_{8t+8}(i-1)|_x = 0$ but this holds by the explicit description obtained for $C_{8t+8}(i-1)$.

Case 4: Let us suppose that

$$|C_{8t}(i)|_\alpha \geq |C_{8t}(i-1)|_\alpha \quad \text{and} \quad |C_{8t}(i)|_\alpha \geq |C_{8t}(i+1)|_\alpha$$

In this case $|C_{8t}(i)|_x = |C_{8t}(i)|_\alpha - |C_{8t}(i+1)|_\alpha \geq 0$, $|C_{8t}(i-1)|_x = |C_{8t}(i)|_y = 0$ and $|C_{8t}(i-1)|_y = |C_{8t}(i)|_\alpha - |C_{8t}(i-1)|_\alpha \geq 0$.

Also, $max\{|C_{8t}(i-1)|_\alpha, |C_{8t}(i)|_\alpha+1, |C_{8t}(i+1)|_\alpha\} = |C_{8t}(i)|_\alpha+1$. We will prove that C_{8t+8} is a representative configuration, $|C_{8t+8}(i)|_\alpha = |C_{8t}(i+1)|_\alpha + 1$ and $|C_{8t+8}(j)|_\alpha = |C_{8t}(j)|_\alpha$ for $j \in \{1,\dots,i-1,i+1,\dots,L\}$

$C_{8t}(i) \xrightarrow{R^i_*} C_{8t+1}(i) = C_{8t}(i) \cup \{c_0\}^+ \xrightarrow{R^i_1} C_{8t+2}(i) = C_{8t}(i) \cup \{c_1\}^+ \xrightarrow{R^i_2} C_{8t+3}(i) = C_{8t}(i) \cup \{c_2\}^+$

$C_{8t+k}(j) = C_{8t}(j)$, for all $j \in \{1,\dots,i-1,i+1,\dots,L\}$ as no rules affect to them and $C_{8t+k}(env) = \emptyset$ for $k \in \{1,2,3\}$.

As cell i has positive electrical charge in C_{8t+3}, then we apply the rule $[\,]_{i-1}, [c_2]^+_i \to [c_{n3}]^-_{i-1}, [\,]_i$ obtaining

$C_{8t+4}(i-1) = C_{8t}(i-1) \cup \{c_{n3}\}^-$

$C_{8t+4}(j) = C_{8t}(j)$ for all $j \in \{1,\dots,i-2,i,\dots,L\}$ and $C_{8t+4}(env) = \emptyset$

After that,

$C_{8t+4}(i-1) = C_{8t}(i-1) \cup \{c_{n3}\}^- \xrightarrow{R^{i-1}_{10}} C_{8t+5}(i-1) = C_{8t}(i-1) \cup \{c_{n4}\}^- \xrightarrow{R^{i-1}_{11}} C_{8t+6}(i-1) = C_{8t}(i-1) \cup \{x\,y\,c_5\}^-$

$C_{8t+k}(j) = C_{8t}(j)$, for all $j \in \{1,\dots,i-2,i,\dots,L\}$ as no rules affect to them and $C_{8t+k}(env) = \emptyset$ for $k \in \{5,6\}$.

At this step rules $[c_5 \to c_6]^-_{i-1}$ and $[x]^-_{i-1}, [\,] \to [\,]_{i-1}[z]_i$ are applied simultaneously and therefore the object x in cell $i-1$ is sent into cell i transformed into an object z. Hence

$C_{8t+7}(i-1) = C_{8t}(i-1) \cup \{y\,c_6\}$

$C_{8t+7}(i) = C_{8t}(i) \cup \{z\}$

$C_{8t+7}(j) = C_{8t}(j)$, for all $j \in \{1,\dots,i-2,i+1,\dots,L\}$ as no rules affect to them and $C_{8t+7}(env) = \emptyset$.

Now, rules $[c_6]_{i-1} \to p\,[\,]_{i-1}$ and $[z \to x\,\alpha]_i$ can be applied, then

$C_{8t+8}(i-1) = C_{8t}(i-1) \cup \{y\}$

$C_{8t+8}(i) = C_{8t}(i) \cup \{x\,\alpha\}$

$C_{8t+8}(j) = C_{8t}(j)$, for all $j \in \{1,\dots,i-2,i+1,\dots,L\}$ as no rules affect to them and $C_{8t+8}(env) = \{p\}$.

Note that $|C_{8t+8}(i)|_\alpha = |C_{8t}(i)|_\alpha + 1$ and that there is no x in $C_{8t+8}(i-1)$ and no $y$in $C_{8t+8}(i)$.

Finally, as in this case $|C_{8t+8}(i-1)|_\alpha = |C_{8t}(i-1)|_\alpha < |C_{8t}(i)|_\alpha + 1 = |C_{8t+8}(i)|_\alpha$ and $|C_{8t+8}(i+1)|_\alpha = |C_{8t}(i+1)|_\alpha < |C_{8t}(i)|_\alpha + 1 = |C_{8t+8}(i)|_\alpha$, then

$$|C_{8t+8}(i-1)|_\alpha < |C_{8t+8}(i)|_\alpha \quad \text{and} \quad |C_{8t+8}(i+1)|_\alpha < |C_{8t+8}(i)|_\alpha$$

so

$$\begin{aligned} |C_{8t+8}(i-1)|_y &\overset{(1)}{=} |C_{8t}(i-1)|_y + 1 \\ &\overset{(2)}{=} |C_{8t}(i)|_\alpha - |C_{8t}(i-1)|_\alpha + 1 \\ &\overset{(3)}{=} |C_{8t+8}(i)|_\alpha - |C_{8t+8}(i-1)|_\alpha \end{aligned}$$

and

$$\begin{aligned} |C_{8t+8}(i)|_x &\overset{(1)}{=} |C_{8t}(i)|_x + 1 \\ &\overset{(2)}{=} |C_{8t}(i)|_\alpha - |C_{8t}(i+1)|_\alpha + 1 \\ &\overset{(3)}{=} |C_{8t+8}(i)|_\alpha - |C_{8t+8}(i+1)|_\alpha \end{aligned}$$

The equality (1) holds by the explicit description obtained for $C_{8t+8}(i)$. The equality (2) holds by hypothesis of induction and the last equality holds by the above reasoning about $|C_{8t+8}(i)|_\alpha$ and because for all $j \in \{1,\dots,L\}$, $i \neq j$, $|C_{8t+8}(j)|_\alpha = |C_{8t}(j)|_\alpha$, in particular for $j = i-1$ and $j = i+1$.

Case 5: Let us suppose that

$$|C_{8t}(i+1)|_\alpha > |C_{8t}(i)|_\alpha \geq |C_{8t}(i-1)|_\alpha$$

In this case $|C_{8t}(i-1)|_x = |C_{8t}(i)|_x = 0$, $|C_{8t}(i-1)|_y = |C_{8t}(i)|_\alpha - |C_{8t}(i-1)|_\alpha \geq 0$, $|C_{8t}(i)|_y = |C_{8t}(i+1)|_\alpha - |C_{8t}(i)|_\alpha > 0$.
Also, $max\{|C_{8t}(i-1)|_\alpha, |C_{8t}(i)|_\alpha + 1, |C_{8t}(i+1)|_\alpha\} = |C_{8t}(i+1)|_\alpha$. We will prove that C_{8t+8} is a representative configuration, $|C_{8t+8}(i)|_\alpha = |C_{8t}(i+1)|_\alpha$ and $|C_{8t+8}(j)|_\alpha = |C_{8t}(j)|_\alpha$ for $j \in \{1,\dots,i-1,i+1,\dots,L\}$
As y is present in cell i,

$C_{8t}(i) \xrightarrow{R^i_*} C_{8t+1}(i) = C_{8t}(i) \cup \{c_0\}^+ \xrightarrow{R^i_1, R^i_3} C_{8t+2}(i) = \{\alpha^{|C_{8t}(i)|_\alpha}\} \cup \{z^{|C_{8t}(i)|_y}\, \alpha^{|C_{8t}(i)|_y}\, c_1\}^+$

$C_{8t+k}(j) = C_{8t}(j)$, for all $j \in \{1,\dots,i-1,i+1,\dots,L\}$ as no rules affect to them and $C_{8t+k}(env) = \emptyset$ for $k \in \{1,2\}$.
As cell i has positive electrical charge in C_{8t+2}, then we apply the rules $[\,]_{i-1}, [z]_i^+ \to [y]_{i-1}, [\,]_i$ and $[c_1 \to c_2]_i^+$ obtaining

$C_{8t+3}(i-1) = C_{8t}(i-1) \cup \{y^{|C_{8t}(i)|_y}\}$
$C_{8t+3}(i) = \{\alpha^{|C_{8t}(i)|_\alpha + |C_{8t}(i)|_y}\, c_2\}$
$C_{8t+3}(j) = C_{8t}(j)$, for all $j \in \{1,\dots,i-2,i+1,\dots,L\}$ and $C_{8t+3}(env) = \emptyset$.

Now, the rule $[\,]_{i-1}, [c_2]_i^+ \to [c_3]_{i-1}^-, [\,]_i$ can be applied. Therefore,

$C_{8t+4}(i-1) = C_{8t}(i-1) \cup \{y^{|C_{8t}(i)|_y}\, c_3\}^-$
$C_{8t+4}(i) = \{\alpha^{|C_{8t}(i)|_\alpha + |C_{8t}(i)|_y}\}$
$C_{8t+4}(j) = C_{8t}(j)$, for all $j \in \{1,\dots,i-2,i+1,\dots,L\}$ and $C_{8t+4}(env) = \emptyset$.

After that,

$C_{8t+4}(i-1) = C_{8t}(i-1) \cup \{y^{|C_{8t}(i)|_y}\, c_3\}^- \xrightarrow{R^{i-1}_6} C_{8t+5}(i-1) = C_{8t}(i-1) \cup \{y^{|C_{8t}(i)|_y}\, c_4\}^- \xrightarrow{R^{i-1}_7} C_{8t+6}(i-1) = C_{8t}(i-1) \cup \{y^{|C_{8t}(i)|_y}\, c_5\}^- \xrightarrow{R^{i-1}_8} C_{8t+7}(i-1) = C_{8t}(i-1) \cup \{y^{|C_{8t}(i)|_y}\, c_6\}^-$

$C_{8t+k}(i) = \{\alpha^{|C_{8t}(i)|_\alpha + |C_{8t}(i)|_y}\}$ and $C_{8t+k}(j) = C_{8t}(j)$, for all $j \in \{1,\dots,i-2,i+1,\dots,L\}$ and $C_{8t+k}(env) = \emptyset$ for $k \in \{5,6,7\}$.

Next, the rule $[c_6]_{i-1} \to p\,[\,]_{i-1}$ is applied,

$C_{8t+8}(i-1) = C_{8t}(i-1) \cup \{y^{|C_{8t}(i)|_y}\}^-$
$C_{8t+8}(i) = \{\alpha^{|C_{8t}(i)|_\alpha + |C_{8t}(i)|_y}\}$ and $C_{8t+8}(j) = C_{8t}(j)$, for all $j \in \{1,\dots,i-2,i+1,\dots,L\}$ and $C_{8t+8}(env) = \{p\}$.

Note that $|C_{8t+8}(i)|_\alpha = |C_{8t}(i)|_\alpha + |C_{8t}(i)|_y$ and by hypothesis $|C_{8t}(i)|_y = |C_{8t}(i+1)|_\alpha - |C_{8t}(i)|_\alpha$ so $|C_{8t+8}(i)|_\alpha = |C_{8t}(i+1)|_\alpha = |C_{8t+8}(i+1)|_\alpha$, and that there is no x or y in $C_{8t+8}(i)$.
Finally, as $|C_{8t+8}(i-1)|_\alpha = |C_{8t}(i-1)|_\alpha < |C_{8t}(i)|_\alpha + |C_{8t}(i)|_y = |C_{8t+8}(i)|_\alpha$, then

$$|C_{8t+8}(i-1)|_\alpha < |C_{8t}(i)|_\alpha$$

and

$$\begin{aligned} |C_{8t+8}(i-1)|_y &\stackrel{(1)}{=} |C_{8t}(i-1)|_y + |C_{8t}(i)|_y \\ &\stackrel{(2)}{=} (|C_{8t}(i)|_\alpha - |C_{8t}(i-1)|_\alpha) + (|C_{8t}(i+1)|_\alpha - |C_{8t}(i)|_\alpha) \\ &\stackrel{(3)}{=} |C_{8t}(i+1)|_\alpha - |C_{8t}(i-1)|_\alpha \\ &\stackrel{(4)}{=} |C_{8t+8}(i)|_\alpha - |C_{8t+8}(i-1)|_\alpha \end{aligned}$$

The equality (1) holds by the explicit description obtained for $C_{8t+8}(i-1)$. The equality (2) holds by hypothesis of induction. The third equality by arithmetic and the last equality holds by the above reasoning about $|C_{8t+8}(i)|_\alpha$ and because for all $j \in \{1, \ldots, L\}$, $i \neq j$, $|C_{8t+8}(j)|_\alpha = |C_{8t}(j)|_\alpha$, in particular for $j = i-1$.
In order to finish the proof that C_{8t+8} is a representative configuration we need to see that $|C_{8t+8}(i-1)|_x = 0$ but this holds by the explicit description obtained for $C_{8t+8}(i-1)$.