The P versus NP problem: Unconventional insights from Membrane Computing

Mario J. Pérez Jiménez

Academia Europaea (The Academy of Europe)
Research Group on Natural Computing
Dpt. of Computer Science and Artificial Intelligence
University of Sevilla, Spain

www.cs.us.es/~marper  marper@us.es

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The P versus NP problem (I)
The P versus NP problem (I)

P = NP?
The $P$ versus $NP$ problem (I)

- Finding solutions versus checking the correctness of solutions.
- Proofs versus verifying their correctness.
The P versus NP problem (I)

- Finding solutions versus checking the correctness of solutions.
- Proofs versus verifying their correctness.

- This is essentially the central problem of Computational Complexity theory.
The P versus NP problem (II)

It is widely believed that it is harder to solve a problem than to check the correctness of a solution. It is widely believed that $P \neq NP$. 
The P versus NP problem (II)

It is widely believed that it is harder

- to solve a problem than to check the correctness of a solution
The \( P \) versus \( NP \) problem (II)

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It is widely believed that \( P \neq NP \).
The P versus NP problem (II)

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It is widely believed that $P \neq NP$. 
Attacking the P versus NP problem

Classical approach (1970):
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- $P \neq NP$.
  - Find an NP-complete problem such that it does not belong to the class $P$. 
Attacking the P versus NP problem

Classical approach (1970):

- $P \neq NP$.
  
  ▶ Find an $NP$-complete problem such that it does not belong to the class $P$.

- $P = NP$.
  
  ▶ Find an $NP$-complete problem such that it belongs to the class $P$. 
Goal:

- Unconventional approaches/tools to attack the **P versus NP problem** are given by using **Membrane Computing**.
Unconventional framework: Membrane Computing

  - It was selected by the Institute for Scientific Information, USA, as a Fast Emerging Research Front in Computer Science (2003).
- The devices of this paradigm (P systems or membrane systems), provide distributed parallel and nondeterministic computing models.
- A computational complexity theory in Membrane Computing is proposed.
  - Polynomial complexity classes associated with (cell–like and tissue–like) P systems are presented.
  - A notion of acceptance must be defined in the new framework (different than the classical one for nondeterministic Turing machines).
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• A **computational complexity theory** in Membrane Computing is proposed.
  ▶ Polynomial complexity classes associated with (cell–like and tissue–like) *P systems* are presented.
    ▶ A notion of *acceptance* must be defined in the new framework (*different* than the classical one for nondeterministic Turing machines)
The notion of acceptance

Nondeterministic Turing machines

Membrane systems
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- Every decision problem has associated a language in a natural way.
Recognizer devices

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- In the real-life, many abstract problems are combinatorial optimization problems not decision problems.

- Every decision problem has associated a language in a natural way.

- The solvability of decision problems is defined through the recognition of the languages associated with them.
Recognizer Membrane Systems

- **Cell-like P systems**: \( \Pi = (\Gamma, \Sigma, H, \mu, M_1, \ldots, M_q, R, i_{in}, i_{out}) \)

- **Tissue-like P systems**: \( \Pi = (\Gamma, \Sigma, \mathcal{E}, M_1, \ldots, M_q, R, i_{in}, i_{out}) \)
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- **Tissue-like P systems:** \( \Pi = (\Gamma, \Sigma, E, M_1, \ldots, M_q, R, i_{in}, i_{out}) \)

  - The working alphabet contains two distinguished elements *yes* and *no*.
  - All computations halt.
  - For any computation of the system, either object *yes* or object *no* (but not both) must have been sent to the output region of the system, and only at the last step of the computation.
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- Accepting/rejecting computations for recognizer P systems
The rules of a membrane system are applied in a nondeterministic maximally parallel manner.
Semantics

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- **Configuration.**
  - *Initial configuration.*
  - *Halting configuration.*
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- Configuration.
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- Computation.
Semantics

The rules of a membrane system are applied in a nondeterministic maximally parallel manner.

- Configuration.
  - Initial configuration.
  - Halting configuration.
- Transition step.
- Computation.
  - Halting computation (accepting or rejecting)
Polynomial time solvability

▶ A decision problem $X$ is *solvable in polynomial time* by a family of recognizer membrane systems $\Pi = \{\Pi(n) : n \in \mathbb{N}\}$, iff:

- The family $\Pi$ is *polynomially uniform by Turing machines*, that is, there exists a DTM working in polynomial time which constructs the system $\Pi(n)$ from $n \in \mathbb{N}$.
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- The family $\Pi$ is *polynomially uniform by Turing machines*, that is, there exists a DTM working in polynomial time which constructs the system $\Pi(n)$ from $n \in \mathbb{N}$.
- There exists a pair $(\text{cod}, s)$ of polynomial-time computable functions over $I_X$ such that:

  - (a) for each instance $u \in I_X$, $s(u)$ is a natural number and $\text{cod}(u)$ is an input multiset of the system $\Pi(s(u))$;
  - (b) for each $n \in \mathbb{N}$, $s(n - 1)$ is a finite set;
  - (c) the family $\Pi$ is polynomially bounded with regard to $(X, \text{cod}, s)$, that is, there exists a polynomial function $p$, such that for each $u \in I_X$ every computation of $\Pi(s(u))$ with input $\text{cod}(u)$ is halting and it performs at most $p(|u|)$ steps;
  - (d) the family $\Pi$ is sound with regard to $(X, \text{cod}, s)$, that is, for each $u \in I_X$, if there exists an accepting computation of $\Pi(s(u))$ with input $\text{cod}(u)$, then $\theta_X(u) = 1$;
  - (e) the family $\Pi$ is complete with regard to $(X, \text{cod}, s)$, that is, for each $u \in I_X$, if $\theta_X(u) = 1$, then every computation of $\Pi(s(u))$ with input $\text{cod}(u)$ is an accepting one.
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  (d) the family $\Pi$ is *sound* with regard to $(X, \text{cod}, s)$, that is, for each $u \in I_X$, if there exists an accepting computation of $\Pi(s(u))$ with input $\text{cod}(u)$, then $\theta_X(u) = 1$;

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We denote it by $X \in \text{PMC}_R$

$\text{PMC}_R$ is closed under complement and polynomial–time reductions.
Solvability of a decision problem

\[ u \in \Sigma^* \]

Input

Polynomial encoding

\[ \Pi(s(u)) \]

Answer of the problem

Answer of the P system

\[ \text{cod}(u) \]

input

\[ \text{cod} \]

\( \Sigma \)
Efficiency of a membrane system

- **Efficiency**: Capability to solve **NP**-complete problems in polynomial time.

\[ \text{NP} \cup \text{co-NP} \subseteq \text{PMC}_R \]

- **Non-Efficiency**: **P** = **PMC**.  

Frontiers of the efficiency:

- **M**\(_1\) efficient.
- **M**\(_2\) non efficient.
- **M**\(_2\) \(\subseteq\) **M**\(_1\): each solution **S** of a problem \(X\) in **M**\(_2\) is also a solution in **M**\(_1\).

Passing from **M**\(_2\) to **M**\(_1\) amounts to passing from non efficiency to efficiency.
Efficiency of a membrane system

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Efficiency of a membrane system

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Managing frontiers of the efficiency

Non efficiency

Efficiency

$M_2$

$M_1$
Attacking the P versus NP problem

- **P = NP**
  - Finding an **NP**-complete problem efficiently solvable in $M_2$.
    - Translating a polynomial time solution of an **NP**-complete problem in $M_1$, to a polynomial time solution in $M_2$.

- **P ≠ NP**
  - Finding an **NP**-complete problem that is not polynomial time solvable in $M_2$. 
Basic cell-like membrane systems

• $\Pi = (\Gamma, \Sigma, H, \mu, M_1, \ldots, M_q, R, i_{in}, i_{out})$. 
Basic cell-like membrane systems

• \( \Pi = (\Gamma, \Sigma, H, \mu, M_1, \ldots, M_q, \mathcal{R}, i_{in}, i_{out}) \).

• Basic transition P systems:
  ▶ \( [u]_h \rightarrow [v]_h \) (evolution rules).
  ▶ \( [u]_h \rightarrow v [ ]_h \) and \( u [ ]_h \rightarrow [v]_h \) (communication rules).
  ▶ \( [u]_h \rightarrow v \) (dissolution rules).

• \( \mathcal{T} \): class of recognizer basic transition P systems.
Efficiency of cell-like membrane systems

- **Proposition 1** (*Sevilla theorem*, 2004)
  Every DTM working in polynomial time can be simulated in polynomial time by a family of recognizer basic transition P systems.

- **Proposition 2** (*Milano theorem*, 2000)
  If a decision problem is solvable in polynomial time by a family of recognizer basic transition P systems with input membrane, then there exists a DTM solving it in polynomial time.
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- **Theorem:** $P = \text{PMC}_T$ *(Sevilla team, 2004).*
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- **Theorem**: \( P = \text{PMC}_T \) *(Sevilla team, 2004).*

  - **Corollary**: \( P \neq \text{NP} \) if and only if every, or at least one, \( \text{NP} \)-complete problem is not in \( \text{PMC}_T \).
P systems with active membranes

- Electrical charges associated with membranes.
- Type of rules:
  
  \[(a) \quad [a \rightarrow u]_{h}^{\alpha} \quad (object \ evolution \ rules).\]

  \[(b) \quad a[\ ]_{h}^{\alpha_1} \rightarrow [b]_{h}^{\alpha_2} \quad (send-in \ communication \ rules).\]

  \[(c) \quad [a]_{h}^{\alpha_1} \rightarrow [\ ]_{h}^{\alpha_2} b \quad (send-out \ communication \ rules).\]

  \[(d) \quad [a]_{h}^{\alpha} \rightarrow b \quad (dissolution \ rules).\]

  \[(e) \quad [a]_{h}^{\alpha_1} \rightarrow [b]_{h}^{\alpha_2} [c]_{h}^{\alpha_3} \quad (division \ rules \ for \ elementary \ membranes).\]

  \[(f) \quad [\ ]_{h_1}^{\alpha_1} [\ ]_{h_2}^{\alpha_2} \rightarrow [\ ]_{h_1}^{\alpha_3} [\ ]_{h_2}^{\alpha_4} \quad (division \ rules \ for \ non-elementary \ membranes).\]

- Non cooperation.
- Non cooperation.
- The sets $\mathcal{NAM}$, $\mathcal{AM}(-ne)$ and $\mathcal{AM}(+ne)$. 
Efficiency of P systems with active membranes

- **Proposition 3:** A deterministic P system with active membranes but **without membrane division** can be simulated by a DTM with a polynomial slowdown.

**Theorem:** $P = PMC_{NAM}$.
Efficiency of P systems with active membranes

- **Proposition 3:** A deterministic P system with active membranes but *without membrane division* can be simulated by a DTM with a polynomial slowdown.

  **Theorem:** $P = PMC_{\mathcal{NAM}}$.

- Efficient solutions to NP–complete problems in $\mathcal{AM}(−ne)$:

  $\Rightarrow$ $NP \cup co-NP \subseteq PMC_{\mathcal{AM}(−ne)}$ (Sevilla team 2003, A. Alhazov, C. Martín and L. Pan, 2004).

- A borderline between efficiency and non–efficiency: division rules in the framework of $\mathcal{AM}(−ne)$.

- Bounds for the complexity class $PMC_{\mathcal{AM}(+ne)}$:

  $PSPACE \subseteq PMC_{\mathcal{AM}(+ne)} \subseteq EXP$ (A.E. Porreca, G. Mauri and C. Zandron, 2006).

- **Conclusion:** $\mathcal{AM}$ is too powerful from the complexity point of view.
Efficiency of P systems with active membranes

- **Proposition 3:** A deterministic P system with active membranes but *without membrane division* can be simulated by a DTM with a polynomial slowdown.

**Theorem:** \( P = \text{PMC}_{\text{NAM}} \).

- Efficient solutions to \( \text{NP} \)-complete problems in \( \text{AM}(\neg ne) \):
  - \( \text{NP} \cup \text{co-NP} \subseteq \text{PMC}_{\text{AM}(\neg ne)} \) (Sevilla team 2003, A. Alhazov, C. Martín and L. Pan, 2004).

- A *borderline* between efficiency and non-efficiency: *division rules* in the framework of \( \text{AM}(\neg ne) \).

\("\text{PMC}_{\text{NAM}}"\)
Efficiency of P systems with active membranes

- **Proposition 3**: A deterministic P system with active membranes but *without membrane division* can be simulated by a DTM with a polynomial slowdown.

  **Theorem**: $P = \text{PMC}_{\mathcal{N},\mathcal{AM}}$.

- Efficient solutions to $\text{NP}$–complete problems in $\mathcal{AM}(-ne)$:
  - $\text{NP} \cup \text{co-NP} \subseteq \text{PMC}_{\mathcal{AM}}(-ne)$ (Sevilla team 2003, A. Alhazov, C. Martín and L. Pan, 2004).

- A *borderline* between efficiency and non–efficiency: *division rules* in the framework of $\mathcal{AM}(-ne)$.

- Bounds for the complexity class $\text{PMC}_{\mathcal{AM}}(+ne)$:
  - $\text{PSPACE} \subseteq \text{PMC}_{\mathcal{AM}}(+ne) \subseteq \text{EXP}$ (A.E. Porreca, G. Mauri and C. Zandron, 2006).
Efficiency of P systems with active membranes

- **Proposition 3**: A deterministic P system with active membranes but *without membrane division* can be simulated by a DTM with a polynomial slowdown.

  **Theorem**: $P = \text{PMC}_{\mathcal{NAM}}$.

- Efficient solutions to NP-complete problems in $\mathcal{AM}(-ne)$:
  
  $\mathsf{NP} \cup \mathsf{co-NP} \subseteq \text{PMC}_{\mathcal{AM}(-ne)}$ (Sevilla team 2003, A. Alhazov, C. Martín and L. Pan, 2004).

- A *borderline* between efficiency and non-efficiency: *division rules* in the framework of $\mathcal{AM}(-ne)$.

- Bounds for the complexity class $\text{PMC}_{\mathcal{AM}(+ne)}$:
  
  $\mathsf{PSPACE} \subseteq \text{PMC}_{\mathcal{AM}(+ne)} \subseteq \mathsf{EXP}$ (A.E. Porreca, G. Mauri and C. Zandron, 2006).

- **Conclusion**: $\mathcal{AM}$ is too powerful from the complexity point of view.
Polarizationless P systems with active membranes

- $\Pi = (\Gamma, \Sigma, H, \mu, M_1, \ldots, M_q, R, i_{in}, i_{out})$:
  
  (a) $[a \rightarrow u]_h$ (object evolution rules).
  
  (b) $a[\ ]_h \rightarrow [b]_h$ (send–in communication rules).
  
  (c) $[a]_h \rightarrow [\ ]_h b$ (send–out communication rules).
  
  (d) $[a]_h \rightarrow b$ (dissolution rules).
  
  (e) $[a]_h \rightarrow [b]_h [c]_h$ (division rules for elementary membranes).
  
  (f) $[[\ ]_h_1 [\ ]_h_2]_h \rightarrow [[[\ ]_h_1]_h [\ ]_h_2]_h$ (division rules for non–elementary membranes).

- The sets $NAM^0, AM^0(\alpha, \beta)$, where $\alpha \in \{-d, +d\}$ and $\beta \in \{-ne, +ne\}$. 
A Păun’s conjecture

At the beginning of 2005, Gh. Păun (problem F from \(^1\)) wrote:

My favorite question (related to complexity aspects in \(P\) systems with active membranes and with electrical charges) is that about the number of polarizations. Can the polarizations be completely avoided? The feeling is that this is not possible – and such a result would be rather sound: passing from no polarization to two polarizations amounts to passing from non–efficiency to efficiency.

The so–called Păun’s conjecture can be formally formulated:

\[
P = \text{PMC}_{\mathcal{AM}^0(+d,−ne)}
\]

Partial answers

AFFIRMATIVE
Partial answers

**AFFIRMATIVE**

- Non efficiency of $\mathcal{A}\mathcal{M}^0(-d, +ne)$

**Theorem:** $P = \text{PMC}_{\mathcal{A}\mathcal{M}^0(-d, +ne)}$ (Sevilla team, 2006).
Partial answers

AFFIRMATIVE

- Non efficiency of $\mathcal{AM}^0(-d, +ne)$

Theorem: $P = \text{PMC}_{\mathcal{AM}^0(-d, +ne)}$ (Sevilla team, 2006).

- The notion of dependency graph.
Partial answers

**AFFIRMATIVE**

- Non efficiency of $\mathcal{AM}^0(-d, +ne)$

**Theorem:** $P = \text{PMC}_\mathcal{AM}^0(-d, +ne)$ (Sevilla team, 2006).

- The notion of dependency graph.

**NEGATIVE**
Partial answers

**AFFIRMATIVE**

- Non efficiency of $\mathcal{AM}^0 (-d, +ne)$

Theorem: $P = \text{PMC}_{\mathcal{AM}^0 (-d, +ne)}$ (Sevilla team, 2006).

- The notion of dependency graph.

**NEGATIVE**

- Efficiency of $\mathcal{AM}^0 (+d, +ne)$:

  - $\text{PSPACE} \subseteq \text{PMC}_{\mathcal{AM}^0 (+d, +ne)}$ (A. Alhazov, P-J, 2007).
Partial answers

**AFFIRMATIVE**

- Non efficiency of $\mathcal{AM}^0(-d,+ne)$

**Theorem:** $P = \text{PMC}_{\mathcal{AM}^0(-d,+ne)}$ (Sevilla team, 2006).

  ▶ The notion of dependency graph.

**NEGATIVE**

- Efficiency of $\mathcal{AM}^0(+d,+ne)$:

  ▶ $\text{PSPACE} \subseteq \text{PMC}_{\mathcal{AM}^0(+d,+ne)}$ (A. Alhazov, P-J, 2007).

A borderline of the efficiency

▶ dissolution rules in $\mathcal{AM}^0(+ne)$. 
On efficiency of polarizationless P systems with active membranes

![Diagram showing the efficiency of polarizationless P systems with active membranes]

- **(−d,+ne)**: Non-Effic
- **(+d,+ne)**: Effic
- **(−d,−ne)**: Non-Effic
- **(+d,−ne)**: Paun’s Conjecture

**Division** and **Dissolution**

- **Without**
- **With**

- **Elementary and non elementary**
- **Only elementary**
Tissue-like membrane systems (I)

- $\Pi = (\Gamma, \Sigma, \mathcal{E}, M_1, \ldots, M_q, R, i_{in}, i_{out})$

- **Basic tissue P systems:**
  - $(i, u/v, j)$, for $i, j \in \{0, 1, \ldots, q\}$, $i \neq j$, and $u, v \in \Gamma^*$ (*symport-antiport rules*).
  - *Length* of the rule $(i, u/v, j)$: $|u| + |v|$.

The set $\mathcal{T}C$. 
Tissue-like membrane systems (II)

- **Tissue P systems with cell division:**
  - Symport-antiport rules.
  - $[a]_i \rightarrow [b]_i[c]_i$, where $i \in \{1, 2, \ldots, q\}$ and $a, b, c \in \Gamma$ (division rules).

- **Tissue P systems with cell separation:**
  - Symport-antiport rules.
  - $[a]_i \rightarrow [\Gamma_1]_i[\Gamma_2]_i$, where $i \in \{1, 2, \ldots, q\}$, $a \in \Gamma$, $i \neq i_{out}$ and $\{\Gamma_1, \Gamma_2\}$ is a fixed partition of $\Gamma$ (separation rules).
Tissue-like membrane systems (II)

- Tissue P systems with cell division:
  - Symport-antiport rules.
  - \([ a ]_i \rightarrow [ b ]_i[ c ]_i\), where \(i \in \{1, 2, \ldots, q\}\) and \(a, b, c \in \Gamma\) (division rules).

- Tissue P systems with cell separation:
  - Symport-antiport rules.
  - \([ a ]_i \rightarrow [ \Gamma_1 ]_i[ \Gamma_2 ]_i\), where \(i \in \{1, 2, \ldots, q\}\), \(a \in \Gamma\), \(i \neq i_{out}\) and \(\{\Gamma_1, \Gamma_2\}\) is a fixed partition of \(\Gamma\) (separation rules).

- The sets \(TDC\), \(TSC\), and \(TDC(k)\), \(TSC(k)\), for each \(k \geq 1\).
Cell Division.
- Cell Division.
- Cell Separation.
Efficiency of tissue P systems

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- $P = \text{PMC}_{TC}$ (Sevilla team, 2009).

- $\text{NP} \cup \text{co-NP} \subseteq \text{PMC}_{TDC}$ (2) (A. Porreca, N. Murphy, P-J, 2012).

- $\text{NP} \cup \text{co-NP} \subseteq \text{PMC}_{TSC}$ (3) (P. Sosík, P-J, 2012).
Efficiency of tissue $P$ systems

- $P = \text{PMC}_{TC}$ (Sevilla team, 2009).
- $P = \text{PMC}_{TDC(1)}$ (Sevilla team, 2010).
Efficiency of tissue P systems

- $P = PMC_{TC}$ (Sevilla team, 2009).
- $P = PMC_{TDC(1)}$ (Sevilla team, 2010).
- $P = PMC_{TSC(2)}$ (L. Pan, P-J, A. Riscos, M. Rius, 2012).

Borderlines of the efficiency

- Division rules in the framework of $TC$.
- Length of communication rules in the framework of $TD$:
  - passing from 1 to 2 amounts to passing from non–efficiency to efficiency.
- Length of communication rules in the framework of $TS$:
  - passing from 2 to 3 amounts to passing from non–efficiency to efficiency.
Efficiency of tissue P systems

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- $\text{NP} \cup \text{co} - \text{NP} \subseteq \text{PMC}_{TSC(3)}$ (P. Sosík, P-J, 2012).
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- **P = PMC_{TC}** (Sevilla team, 2009).
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**Borderlines of the efficiency**

- division rules in the framework of TC.
Efficiency of tissue P systems

- $P = \text{PMC}_{TC}$ (Sevilla team, 2009).
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- $P = \text{PMC}_{TSC(2)}$ (L. Pan, P-J, A. Riscos, M. Rius, 2012).
- $\text{NP} \cup \text{co} - \text{NP} \subseteq \text{PMC}_{TDC(2)}$ (A. Porreca, N. Murphy, P-J, 2012).
- $\text{NP} \cup \text{co} - \text{NP} \subseteq \text{PMC}_{TSC(3)}$ (P. Sosík, P-J, 2012).

**Borderlines of the efficiency**

- division rules in the framework of $TC$.
- length of communication rules in the framework of $TD$: passing from 1 to 2 amounts to passing from non-efficiency to efficiency.
Efficiency of tissue P systems

- \( P = \text{PMC}_{TC} \) (Sevilla team, 2009).
- \( P = \text{PMC}_{TD}(1) \) (Sevilla team, 2010).
- \( P = \text{PMC}_{TS}(2) \) (L. Pan, P-J, A. Riscos, M. Rius, 2012).
- \( \text{NP} \cup \text{co} - \text{NP} \subseteq \text{PMC}_{TD}(2) \) (A. Porreca, N. Murphy, P-J, 2012).
- \( \text{NP} \cup \text{co} - \text{NP} \subseteq \text{PMC}_{TS}(3) \) (P. Sosík, P-J, 2012).

Borderlines of the efficiency

- division rules in the framework of \( TC \).
- length of communication rules in the framework of \( TD \): passing from 1 to 2 amounts to passing from non–efficiency to efficiency.
- length of communication rules in the framework of \( TS \): passing from 2 to 3 amounts to passing from non–efficiency to efficiency.
Efficiency of tissue P systems
Efficiency of tissue P systems
Tissue P systems without environment

• Tissue-like P systems: \( \Pi = (\Gamma, \Sigma, E, M_1, \ldots, M_q, R, i_{\text{in}}, i_{\text{out}}) \)

▶ The objects of \( E \) initially appear located in the environment in an arbitrary number of copies.

• Tissue-like P systems without environment: \( E = \emptyset \).

• The classes \( \hat{\text{TC}}, \hat{\text{TDC}}, \hat{\text{TSC}}, \) and \( \hat{\text{TC}}(k), \hat{\text{TDC}}(k), \hat{\text{TSC}}(k) \), for each \( k \geq 1 \).
Tissue P systems without environment

- **Tissue-like P systems:** \( \Pi = (\Gamma, \Sigma, E, M_1, \ldots, M_q, R, i_{in}, i_{out}) \)
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- **Tissue-like P systems without environment:** \( E = \emptyset \).

- The classes \( \hat{TC}, \hat{TDC}, \hat{TSC}, \) and \( \hat{TC}(k), \hat{TDC}(k), \hat{TSC}(k), \) for each \( k \geq 1 \).
Efficiency of tissue P systems without environment

Division rules

• For each $k$:
  $$PMC_{\hat{TDC}}(k + 1) = PMC_{TDC}(k + 1)$$
  (Sevilla team, 2012).

  $\Rightarrow P = PMC_{\hat{TDC}}(1)$.

  $\Rightarrow NP \cup co-NP \subseteq PMC_{\hat{TDC}}(2)$.

Separation rules

• $P = PMC_{\hat{TSC}}$ (Sevilla team, 2013).

  $\Rightarrow P = PMC_{\hat{TSC}}(3)$.

  $\Rightarrow NP \cup co-NP \subseteq PMC_{TSC}(3)$.

Borderlines of the efficiency

$\Rightarrow$ The environment in the framework $\hat{TSC}(3)$. 
Efficiency of tissue P systems without environment

Division rules

- For each $k$: $\text{PMC}_{TD(k+1)} = \text{PMC}_{TD(k+1)}$ (Sevilla team, 2012).

Separation rules

- $\text{P} = \text{PMC}_{TSC}$ (Sevilla team, 2013).

Borderlines of the efficiency

- $\text{NP} \cup \text{co-NP} \subseteq \text{PMC}_{TDC(k+1)}$. (Sevilla team, 2012).
Efficiency of tissue P systems without environment

Division rules

- For each $k$: $\text{PMC}^{\text{TDC}(k+1)} = \text{PMC}^{\text{TDC}(k+1)}$ (Sevilla team, 2012).
  
  ▶ $P = \text{PMC}^{\text{TDC}(1)}$. 

Separation rules

- $P = \text{PMC}^{\text{TSC}}$ (Sevilla team, 2013).
  
  ▶ $P = \text{PMC}^{\text{TSC}}$.

Borderlines of the efficiency

▶ The environment in the framework $\text{TSC}^{(3)}$. 

Efficiency of tissue P systems without environment

Division rules

- For each $k$: $\text{PMC}_{\overline{TDC}(k+1)} = \text{PMC}_{TDC(k+1)}$ (Sevilla team, 2012).
  - $P = \text{PMC}_{\overline{TDC}(1)}$.
  - $\text{NP} \cup \text{co-NP} \subseteq \text{PMC}_{\overline{TDC}(2)}$.

Separation rules

- $P = \text{PMC}_{\overline{TDC}(1)}$.
- $\text{NP} \cup \text{co-NP} \subseteq \text{PMC}_{\overline{TDC}(2)}$. 

Borderlines of the efficiency

- The environment in the framework $\overline{TDC}(3)$. 

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Division rules

- For each $k$: $\text{PMC}_{\widehat{TDC}(k+1)} = \text{PMC}_{TDC(k+1)}$ (Sevilla team, 2012).
  - $P = \text{PMC}_{\widehat{TDC}(1)}$.
  - $\text{NP} \cup \text{co-NP} \subseteq \text{PMC}_{\widehat{TDC}(2)}$.

- The length of communication rules provides a new borderline of the efficiency in the framework $\widehat{TDC}$. 
Efficiency of tissue P systems without environment

**Division rules**

- For each $k$: \( \text{PMC}_{TD^{C}(k+1)} = \text{PMC}_{TD^{C}(k+1)} \) (Sevilla team, 2012).
  
  - \( P = \text{PMC}_{TD^{C}(1)} \).
  
  - \( NP \cup co - NP \subseteq \text{PMC}_{TD^{C}(2)} \).

- The length of communication rules provides a new borderline of the efficiency in the framework \( TD \).

**Separation rules**

- \( P = \text{PMC}_{TS^{C}} \) (Sevilla team, 2013).
Efficiency of tissue P systems without environment

Division rules

- For each $k$: $\overline{\text{PMC}_{TD\bar{C}}(k+1)} = \overline{\text{PMC}_{TD\bar{C}}(k+1)}$ (Sevilla team, 2012).
  - $P = \overline{\text{PMC}_{TD\bar{C}}(1)}$.
  - $\overline{NP \cup co - NP} \subseteq \overline{\text{PMC}_{TD\bar{C}}(2)}$.
- The length of communication rules provides a new borderline of the efficiency in the framework $\overline{T\bar{D}}$.

Separation rules

- $P = \overline{\text{PMC}_{TS\bar{C}}}$ (Sevilla team, 2013).
  - $P = \overline{\text{PMC}_{TS\bar{C}}(3)}$. 
Efficiency of tissue P systems without environment

Division rules

- For each $k$: $\text{PMC}_{\text{TDC}(k+1)} = \text{PMC}_{\text{TDC}(k+1)}$ (Sevilla team, 2012).
  - $P = \text{PMC}_{\text{TDC}(1)}$.
  - $\text{NP} \cup \text{co} - \text{NP} \subseteq \text{PMC}_{\text{TDC}(2)}$.

- The length of communication rules provides a new borderline of the efficiency in the framework $\text{T\bar{D}}$.

Separation rules

- $P = \text{PMC}_{\text{TSC}}$ (Sevilla team, 2013).
  - $P = \text{PMC}_{\text{TSC}(3)}$.
  - $\text{NP} \cup \text{co} - \text{NP} \subseteq \text{PMC}_{\text{TSC}(3)}$. 
Efficiency of tissue P systems without environment

**Division rules**

- For each $k$: $\text{PMC}_{\overrightarrow{TDC}(k+1)} = \text{PMC}_{\overrightarrow{TDC}(k+1)}$ (Sevilla team, 2012).
  
  $\Rightarrow P = \text{PMC}_{\overrightarrow{TDC}(1)}$.
  
  $\Rightarrow \text{NP} \cup \text{co} \cap \text{NP} \subseteq \text{PMC}_{\overrightarrow{TDC}(2)}$.

- The length of communication rules provides a new borderline of the efficiency in the framework $\overrightarrow{T D}$.

**Separation rules**

- $P = \text{PMC}_{\overrightarrow{TSC}}$ (Sevilla team, 2013).
  
  $\Rightarrow P = \text{PMC}_{\overrightarrow{TSC}(3)}$.
  
  $\Rightarrow \text{NP} \cup \text{co} \cap \text{NP} \subseteq \text{PMC}_{\overrightarrow{TSC}(3)}$.

**Borderlines of the efficiency**

- The environment in the framework $\overrightarrow{TSC}(3)$.
Conclusions: New Frontiers of the efficiency

- Kind of the rules:
  - Division rules in AM
  - Division rules in TC
  - Dissolution rules in AM
- The length of communication rules:
  - Passing from 1 to 2 in TD
  - Passing from 1 to 2 in \( \hat{TD} \)
  - Passing from 2 to 3 in TS
- The environment:
  - In the framework TSC (3.)

Each of them provides a new way to attack the P versus NP problem.
Conclusions: New Frontiers of the efficiency

- **Kind of the rules:**
  - Division rules in $\mathcal{AM}$.
  - Division rules in $\mathcal{TC}$.
  - Dissolution rules in $\mathcal{AM}^0(+ne)$. 

- The length of communication rules:
  - Passing from 1 to 2 in $\hat{\mathcal{TD}}$.
  - Passing from 1 to 2 in $\mathcal{TSC}(3)$.
  - Passing from 2 to 3 in $\mathcal{TS}$.

- The environment:
  - In the framework $\mathcal{TSC}(3)$. Each of them provides a new way to attack the P versus NP problem.
Conclusions: New Frontiers of the efficiency

- **Kind of the rules:**
  - Division rules in $AM$.
  - Division rules in $TC$.
  - Dissolution rules in $AM^0(+ne)$.

- **The length of communication rules:**
  - Passing from 1 to 2 in $TD$.
  - Passing from 1 to 2 in $\overline{TD}$.
  - Passing from 2 to 3 in $TS$. 
Conclusions: New Frontiers of the efficiency

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  - Division rules in $AM$.
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  - Passing from 2 to 3 in $TS$.

- **The environment:**
  - In the framework $TSC(3)$. 

Conclusions: New Frontiers of the efficiency

- **Kind of the rules:**
  - Division rules in $AM$.
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  - Passing from 1 to 2 in $\overline{TD}$.
  - Passing from 2 to 3 in $TS$.

- **The environment:**
  - In the framework $TSC(3)$.

Each of them provides a new way to attack the P versus NP problem.
THANK YOU

FOR YOUR ATTENTION!
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