A polynomial complexity class in P systems using membrane division

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Abstract

In this paper we introduce the complexity class $\mathbf{PMC}_{\mathcal{F}}$ of all decision problems solvable in polynomial time by a family of P systems belonging to a prefixed class of membrane systems with input, $\mathcal{F}$. We show that the problem of determining if a boolean formula in conjunctive normal form is satisfiable, belongs to $\mathbf{PMC}_{\mathcal{AM}}$, being $\mathcal{AM}$ the class of recognizer P systems with input and with active membranes using 2-division. We conclude that the class **NP** is contained in the above complexity class.

1 Introduction

In October 1998 [1] an unconventional model of computation, called *P systems*, within the framework of *Natural Computing*, is introduced by G. Păun as a class of distributed parallel computing model. It comes from the observation that certain processes which take place in the complex structure of living organisms can be considered as computations.

A *complexity class* within a model of computation is a collection of problems that can be solved by some devices of the model with *similar* computational resources. In this paper we present a *polynomial complexity class* in cellular computing with membranes inspired in some ideas of G. Păun ([2], section 7.1) discussed with the authors. These classes allow us to detect some intrinsic difficulties for the resolution of a problem in the model of cellular computing with membranes.

A precise definition of those parameters that specify a complexity class is required: the model and the mode of computation (cellular computing with membranes); the computational resource (time); the bound for these resources (a computable total function mapping natural numbers to natural numbers); and, of course, we need to define what kind of problems will be considered (decision problems).

The paper is organized as follows: in section 2 we define the notion of recognizer P system (working with symbol–objects), which will be the model considered to study the complexity classes; in section 3 the polynomial complexity class in computing with membranes, $\mathbf{PMC}_{\mathcal{AM}}$, is introduced. Section 4 provides a detailed study of the (cellular) complexity of the **SAT** problem; and, finally, in section 5 some conclusions and two open questions are presented.

References

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