Computationally Hard Problems Addressed through P Systems

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Abstract. In this paper we present a general framework to provide efficient solutions to decision problems through families of cell-like membrane systems constructed in a semi–uniform (associating with *each instance* of the problem one P system solving it) or uniform (all instances of a decision problem having the same *size* are processed by the same system) way. Also we show a brief compendium of efficient semi–uniform and uniform solutions to hard problems in these systems, and we explicitely describe some of these solutions.

1 Introduction

Many interesting problems of the real world are presumably intractable (unless **P** = **NP**) and hence it is not possible to execute algorithmic solutions in an electronic computer when we use instances of those problems whose size is *huge*.

Membrane systems have two main ingredients: their inherent parallelism and non-determinism. Can this parallelism and non-determinism be used to solve hard problems in a feasible time? The answer is affirmative, but we must point out two considerations. On one hand, we have to deal with the non-determinism in such a way that the solutions obtained from these devices are algorithmic solutions in the classic sense; that is, the answers of the computations of the system must be reliable. On the other hand, the drastic decrease of the execution time from an exponential to a polynomial one is not achieved for free, but by the use of an exponential amount of space, although this space is created in polynomial time.

In this paper we present the theoretical requirements for a P system to provide an *algorithmic solution* to an abstract decision problem (a precise definition of the latter is given in section 2). First, all the computations of the system must halt, providing an affirmative or negative answer to the problem (for a particular instance of it). Second, we impose the systems to be confluent. This is a generalization of the notion of determinism for which we require all the possible computations to provide the same answer. This way we do not obtain a contradictory result. In section 3 we call recognizer P systems to the systems verifying these two properties.

It is important to note that all the feasible solutions to hard problems by means of these biologically inspired devices presented until now do not use a single P system, but a family of them. However, there are significative differences between those solutions, dividing them in two groups: the *semi–uniform* solutions which associate with *each instance* of the problem one P system solving it; and the *uniform* solutions which associate with *each possible size* of the instances of the problem one P system that can solve all the instances of that size. A formal definition of these two concepts can be found in sections 4 and 5.

Another possible classification can be considered attending to the existence or not in the system of a membrane where to introduce input data before the computation starts. Usually, the semi–uniform solutions are performed by P systems without input, whereas the uniform solutions are performed by P systems with input. In section 4 we present a compendium of known semi–uniform solutions to hard problems by P systems without input, and a detailed description of two of these solutions: one to the *Satisfiability Problem* and the other to the *Hamiltonian Path Problem.* Finally, in section 5 we do the same for known uniform solutions to hard problems by P systems with input, detailing the ones corresponding to the *Decision Knapsack Problem (0/1)* and to the *Common Algorithmic Decision Problem.*

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