Exercise Sheet 3: Semantic tableaux for Propositional Logic

Lógica Informática – Docencia en Inglés

Exercise 1. Let $F$ be the propositional formula $p \land q \land \neg r$.

1. Construct a complete semantic tableau for $F$.
2. Construct a complete semantic tableau for $\neg F$.
3. Convert $F$ into DNF and CNF.

Exercise 2. A semantic tableau for a propositional formula $B$ has exactly two open branches. The first one is labeled by $p, \neg q$ and the second one is labeled by $p, r$. Calculate a CNF of $B$. Simply the normal form as much as possible.

Exercise 3. A semantic tableau for a formula $A$ has an open branch labeled by $p$ and, perhaps, some other open branches. Decide whether the following assertions are true. Justify your answers.

\[
\begin{align*}
A &\models p \\
p &\models A \\
A &\models p \land q \\
p \land q &\models A \\
p \rightarrow A &\in \text{TAUT}
\end{align*}
\]

Exercise 4. Use a semantic tableau to obtain all the models of the following set of formulas.

1. \{\(p \leftrightarrow q, \neg(q \leftrightarrow r), r\}\}
2. \{\(p \rightarrow q, q \rightarrow r, (\neg p \leftrightarrow r)\}\}

Exercise 5. Using the semantic tableaux method, determine whether the following formulas are valid. If they are not valid, give a counter model.

1. $A \land B \rightarrow A \lor B$
2. $A \lor B \rightarrow A \land B$
3. $(A \land B \rightarrow C) \rightarrow (A \rightarrow C) \land (B \rightarrow C)$
4. $(A \land B \rightarrow C) \rightarrow (A \rightarrow C) \lor (B \rightarrow C)$
5. $(A \rightarrow C) \lor (B \rightarrow C) \rightarrow (A \lor B \rightarrow C)$
6. $(A \rightarrow C) \lor (B \rightarrow C) \rightarrow (A \land B \rightarrow C)$

Exercise 6. Check if the following logical consequences hold using the semantic tableaux method. For those that do not hold, exhibit a counter model.

1. \{\(A \rightarrow B, B \lor C \rightarrow D\)\} \models A \rightarrow C
2. \{\(A \rightarrow B, B \land C \rightarrow D\)\} \models A \rightarrow C
3. $A \rightarrow (B \rightarrow C) \models (A \rightarrow B) \rightarrow C$
4. $(A \rightarrow B) \rightarrow C \models A \rightarrow (B \rightarrow C)$

Exercise 7. Formalize the following argument and decide whether or not it is correct by using a semantic tableau.

"I'll go to the movies unless it rains and I feel tired. It is raining. For, if I don't feel tired then I'll go to the movies."