

Ejercicio 7.1. Prueba mediante deducción natural:

1. $\forall x.(P(x) \rightarrow Q(x)) \vdash \forall x.P(x) \rightarrow \forall x.Q(x)$
2. $\exists x.\neg P(x) \vdash \neg \forall x.P(x)$
3. $\forall x.P(x) \vdash \forall y.P(y)$
4. $\forall x.(P(x) \rightarrow Q(x)) \vdash (\forall x.\neg Q(x)) \rightarrow (\forall x.\neg P(x))$
5. $\forall x.(P(x) \rightarrow \neg Q(x)) \vdash \neg(\exists x.(P(x) \wedge Q(x)))$
6. $\forall x.\forall y.P(x, y) \vdash \forall u.\forall v.P(u, v)$
7. $\exists x.\exists y.P(x, y) \vdash \exists u.\exists v.P(u, v)$
8. $\exists x.\forall y.P(x, y) \vdash \forall y.\exists x.P(x, y)$
9. $\exists x.(P(a) \rightarrow Q(x)) \vdash P(a) \rightarrow \exists x.Q(x)$
10. $P(a) \rightarrow \exists x.Q(x), \text{actual } i \vdash \exists x.(P(a) \rightarrow Q(x))$
11. $\exists x.P(x) \rightarrow Q(a) \vdash \forall x.(P(x) \rightarrow Q(a))$
12. $\forall x.(P(x) \rightarrow Q(a)), \text{actual } i \vdash \exists x.(P(x) \rightarrow Q(a))$
13. $\forall x.P(x) \vee \forall x.Q(x) \vdash \forall x.(P(x) \vee Q(x))$
14. $\exists x.(P(x) \wedge Q(x)) \vdash \exists x.P(x) \wedge \exists x.Q(x)$
15. $\forall x.\forall y.(P(y) \rightarrow Q(x)) \vdash \exists y.P(y) \rightarrow \forall x.Q(x)$
16. $\neg \forall x.\neg P(x), \text{actual } j \vdash \exists x.P(x)$
17. $\forall x.\neg P(x) \vdash \neg \exists x.P(x)$
18. $\exists x.P(x) \vdash \neg \forall x.\neg P(x)$
19. $P(a) \rightarrow \forall x.Q(x) \vdash \forall x.(P(a) \rightarrow Q(x))$
20. $\forall x.\forall y.\forall z.(R(x, y) \wedge R(y, z) \rightarrow R(x, z)), \forall x.\neg R(x, x) \vdash \forall x.\forall y.(R(x, y) \rightarrow \neg R(y, x))$
21. $\forall x.(P(x) \vee Q(x)), \exists x.\neg Q(x), \forall x.(R(x) \rightarrow \neg P(x)) \vdash \exists x.\neg R(x)$
22. $\forall x.(P(x) \rightarrow (Q(x) \vee R(x))), \neg \exists x.(P(x) \wedge R(x)) \vdash \forall x.(P(x) \rightarrow Q(x))$
23. $\exists x.\exists y.(R(x, y) \vee R(y, x)) \vdash \exists x.\exists y.R(x, y)$

Ejercicio 7.2. (resueltos en clase) Prueba mediante deducción natural:

1. $P(c), (\forall x)(P(x) \rightarrow \neg Q(x)) \vdash \neg Q(c).$
2. $(\forall x)(P(x) \rightarrow Q(x)), (\forall x)P(x) \vdash (\forall x)Q(x)$
3. $(\forall x)P(x) \vdash (\exists x)P(x)$

4. $(\forall x)(P(x) \rightarrow Q(x)), (\exists x)P(x) \vdash (\exists x)Q(x)$
5. $(\forall x)(Q(x) \rightarrow R(x)), (\exists x)(P(x) \wedge Q(x)) \vdash (\exists x)(P(x) \wedge R(x))$
6. $(\exists x)P(x), (\forall x)(\forall y)(P(x) \rightarrow Q(y)) \vdash (\forall y)Q(y)$
7. $\neg(\forall x)F \equiv (\exists x)\neg F$
8. $(\forall x)(F \wedge G) \equiv (\forall x)F \wedge (\forall x)G$
9. $(\exists x)F \vee (\exists x)G \equiv (\exists x)(F \vee G)$
10. $(\exists x)(\exists y)F \equiv (\exists y)(\exists x)F$