

# TEMA VI

## Modelos probabilísticos basados en sistemas P: Aplicación a un ecosistema real

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# Caso de estudio: nivel macro

## Ecosistema real del quebrantahuesos<sup>1</sup>



- ▶ Especie en peligro de extinción.
- ▶ Zona pirenaico–catalana (datos desde 1994 hasta 2008)<sup>2</sup>

<sup>1</sup> M. Cardona, M.A. Colomer, M.J. Pérez, D. Sanuy, A. Margalida. A bio-inspired computing model as a new tool for modeling ecosystems: The avian scavengers as a case study. *Ecological Modelling*, **1** (2011), 33-47.

<sup>2</sup> M. Cardona, M.A. Colomer, M.J. Pérez-Jiménez, D. Sanuy, A. Margalida. Modelling ecosystem using P systems: The bearded vulture, a case study. *Lecture Notes in Computer Science*, **5391** (2009), 137–156.

El ecosistema real a modelizar está situado en el Pirineo Catalán.



Distribución regional del Quebrantahuesos en el Pirineo Catalán

La vida de distintas especies animales están interrelacionadas a través de dependencias en términos de comida, territorios, etc.

En los ecosistemas montañosos existe una relación tradicional entre ungulados salvajes y sus predadores que ha sido alterada por la presencia de animales domésticos.

El abandono de animales muertos es una fuente de comida de animales necrófagos.

- ▶ Este es el caso de los **quebrantahuesos** (*bearded vultures*, *Gypaetus barbatus*)
  - ▶ una especie que se alimenta de los huesos de animales domésticos y ungulados salvajes.

Punto de interés para el conservacionismo de especies animales:

- ▶ Estudio de la interacción de las especies entre sí y con el entorno.
- ▶ Necesidad de usar modelos matemáticos/computacionales para representar, analizar y formular medidas que ayude al desarrollo sostenible del ecosistema objeto de estudio.

Vamos a diseñar un modelo computacional (basado en computación celular con membranas) que estudia la evolución de un ecosistema real localizado en los Pirineos

## El ecosistema incluye seis especies:

- ▶ **Quebrantahuesos** (*Bearded Vulture: Gypaetus barbatus*)
- ▶ **Rebeco** (*Pyrenean Chamois: Rupicapra pyrenaica*).
- ▶ **Ciervo** (*Red Deer: Cervus elaphus*).
- ▶ **Gamo** (*Fallow Deer: Dama dama*).
- ▶ **Corzo** (*Roe Deer: Capreolus capreolus*).
- ▶ **Oveja** (*Sheep: Ovis capra*).



Las “presas” (animales domésticos y ungulados salvajes) corresponden a la familia de los bóvidos: son herbívoros y sus huesos constituyen la principal fuente de alimentación de los quebrantahuesos.

# Los quebrantahuesos (I)

Es una **especie carroñera** cuyo habitat radica en las montañas de Europa, Asia y Africa.

En 2007 había contabilizadas unas 150 parejas en todo el mundo.

- ▶ Tienen una vida media de 21'4 años.
- ▶ Edad media de la primera reproducción: 8.1 años.
- ▶ La tasa de fertilidad (en Cataluña) se estima en el 38%.
- ▶ Los huevos se incuban desde diciembre a febrero y los polluelos nacen tras 52–54 días de incubación.
  - ▶ Usualmente ponen dos huevos, pero sólo un polluelo sobrevive.
- ▶ A los 120 días abandonan el nido para llevar una vida nómada.
  - ▶ La superficie media recorrida por jóvenes monitorizados fue de 4932  $km^2$  durante ese periodo.
- ▶ A los 8 años ocupan un territorio fijo.
- ▶ Son los únicos vertebrados que se alimentan casi exclusivamente de huesos.
- ▶ Una pareja necesita en torno a 341 kgs. de huesos cada año

# Los quebrantahuesos (II)

La conducta de las 5 especies de bóvidos es similar:

- ▶ Todos son herbívoros.
- ▶ Alcanza su tamaño adulto a la edad de 1 año.
- ▶ Alcanza su madurez sexual a los 2 años.
- ▶ La tasa de mortalidad de dichas especies son similares:
  - ▶ Durante el primer año de vida: en torno al 50 %.
  - ▶ El resto, en torno al 6 %.
- ▶ Los ciervos son muy apreciados por los cazadores:
  - ▶ Por ello, se distinguirán en el modelo entre machos y hembras.
- ▶ Estas 5 especies proporcionan el 67% de la comida que necesitan los quebrantahuesos.
- ▶ En Cataluña existen 7 estaciones de alimentación que proporcionan artificialmente unos 10.500 kgs de huesos por año.

## Los quebrantahuesos (III)

- ▶ Cuando uno de estos animales mueren, el peso de los huesos que dejan disponibles para los quebrantahuesos es, aproximadamente, el 20% del total

Table: Descriptive variables used to model the ecosystem

Specie	Weigh Male kg	Weigh Female kg	Percentage Female	Average weigh kg	Biomass: bone adult kg	Biomass: bone young kg	Kg accessible by B. Vulture (adult/young)
Bearded Vulture	5	6.5	60	5.75	-	-	-
Chamois	28	32	50	30	6	3	6/3
Red Deer Female	-	75	-	75	15	7.5	15/7.5
Red Deer Male	120	-	-	120	24	12	24/12
Fallow Deer	63	42	80	46	9	4.5	2/1
Roe Deer	27	23	66	24	5	2.5	1/0.5
Sheep	42	35	97	35.2	7	3.5	7/3.5

# Una máquina celular que modeliza un ecosistema real (I)

Un PDP system de grado (2, 1):  $(G, \Gamma, \Sigma, \mathcal{R}_\Pi, \mathcal{R}_E, \{f_r : r \in \mathcal{R}_\Pi\}, \mathcal{M}_1, \mathcal{M}_2)$

- ▶  $G = (V, E)$  con  $V = \{e_1\}$  y  $E = \{(e_1, e_1)\}$
- ▶ Working alphabet:  $\Gamma = \{X_{ij}, Y_{ij}, Z_{ij} : 1 \leq i \leq 7, 0 \leq j \leq k_{i,4}\} \cup \{B, C\}$ 
  - ▶  $X, Y, Z$  represent the same animal but in different states.
  - ▶ Index  $i$  is associated with the species.
  - ▶ Index  $j$  is associated with their age.
  - ▶  $B$  represents 0.5 kgs of bones.
  - ▶  $C$  is used for technical reasons.
- ▶  $\Sigma = \mathcal{R}_E = \emptyset$ .
- ▶  $\Pi = (\Gamma, \mu, \mathcal{M}_1, \mathcal{M}_2, \mathcal{R}_\Pi)$ , where:
  - ▶ Structure of membranes:  $\mu = [ [ ]_2 ]_1$ : The skin region where animals reproduce and an inner membrane where animals feed, and die.
  - ▶  $\mathcal{M}_1 = \{X_{ij}^{q_{ij}} : 1 \leq i \leq 7, 0 \leq j \leq k_{i,4}\}$ ,  $q_{ij}$  is the number of animals of species  $i$  with age  $j$ .
  - ▶  $\mathcal{M}_2 = \{C B^\alpha\}$ , where  $\alpha = \lceil \sum_{j=1}^{21} q_{1j} \cdot 1.10 \cdot 682 \rceil$ 
    - $\alpha$  represents an external contribution of food which is added during the first year of study so that the Bearded Vulture survives.
    - Constant 1.10: represents 10% of the population growth.
    - Constant 682: the amount of food needed per year for a Bearded Vulture couple to survive.

# Reglas de Reproducción

Adult males:

$$\blacktriangleright r_0 \equiv [X_{ij} \xrightarrow{(1-k_{i,13}) \cdot (1-k_{i,15})} Y_{ij}]_1, \quad 1 \leq i \leq 7, k_{i,2} \leq j \leq k_{i,4}.$$

Adult females that reproduce:

$$\blacktriangleright r_1 \equiv [X_{ij} \xrightarrow{k_{i,5} \cdot k_{i,13} \cdot (1-k_{i,15})} Y_{ij} Y_{i0}]_1, \quad 1 \leq i \leq 7, k_{i,2} \leq j < k_{i,3}.$$

Fertile adult females that do not reproduce:

$$\blacktriangleright r_2 \equiv [X_{ij} \xrightarrow{(1-k_{i,5}) \cdot k_{i,13} \cdot (1-k_{i,15})} Y_{ij}]_1, \quad 1 \leq i \leq 7, k_{i,2} \leq j < k_{i,3}.$$

Not fertile adult females:

$$\blacktriangleright r_3 \equiv [X_{ij} \xrightarrow{k_{i,13} \cdot (1-k_{i,15})} Y_{ij}]_1, \quad 1 \leq i \leq 7, k_{i,3} \leq j \leq k_{i,4}.$$

Young animals that do not reproduce:

$$\blacktriangleright r_4 \equiv [X_{ij} \xrightarrow{1-k_{i,15}} Y_{ij}]_1, \quad 1 \leq i \leq 7, 0 \leq j < k_{i,2}.$$

# Reglas de Crecimiento

$$\blacktriangleright r_5 \equiv [X_{ij} \xrightarrow{(k_{i,6}+k_{i,10}) \cdot k_{i,15}} Y_{ik_{i,2}} Y_{ij}]_1, \quad 1 \leq i \leq 7, k_{i,2} \leq j < k_{i,4}.$$

$$\blacktriangleright r_6 \equiv [X_{ij} \xrightarrow{(1-k_{i,6}-k_{i,10}) \cdot k_{i,15}} Y_{ij}]_1, \quad 1 \leq i \leq 7, k_{i,2} \leq j < k_{i,4}.$$

$$\blacktriangleright r_7 \equiv [X_{ij} \xrightarrow{k_{i,6} \cdot k_{i,15}} Y_{ik_{i,2}} Y_{ij}]_1, \quad 1 \leq i \leq 7, j = k_{i,4}.$$

$$\blacktriangleright r_8 \equiv [X_{ij} \xrightarrow{(1-k_{i,6}) \cdot k_{i,15}} Y_{ij}]_1, \quad 1 \leq i \leq 7, j = k_{i,4}.$$

# Reglas de Mortalidad

- Young animals.

Those which survive:

$$\blacktriangleright r_9 \equiv Y_{ij}[ ]_2 \xrightarrow{1-k_{i,7}-k_{i,8}} [Z_{ij}]_2 : 1 \leq i \leq 7, 0 \leq j < k_{i,1}.$$

Those which die:

$$\blacktriangleright r_{10} \equiv Y_{ij}[ ]_2 \xrightarrow{k_{i,8}} [B^{k_{i,11}}]_2 : 1 \leq i \leq 7, 0 \leq j < k_{i,1}.$$

Those which are retired of the ecosystem:

$$\blacktriangleright r_{11} \equiv [Y_{ij} \xrightarrow{k_{i,7}} \lambda]_1 : 1 \leq i \leq 7, 0 \leq j < k_{i,1}.$$

- Adult animals that don't arrive at an average life expectancy:

Those which survive:

$$\blacktriangleright r_{12} \equiv Y_{ij}[ ]_2 \xrightarrow{1-k_{i,10}} [Z_{ij}]_2 : 1 \leq i \leq 7, k_{i,1} \leq j < k_{i,4}.$$

Those which die:

$$\blacktriangleright r_{13} \equiv Y_{ij}[ ]_2 \xrightarrow{k_{i,10}} [B^{k_{i,12}}]_2 : 1 \leq i \leq 7, k_{i,1} \leq j < k_{i,4}.$$

- Animals that arrive at an average life expectancy:

Those which growth population depend on the fertility ratio and die in the ecosystem:

$$\blacktriangleright r_{14} \equiv Y_{ij}[ ]_2 \xrightarrow{(1-k_{i,15}) \cdot (k_{i,9} + (1-k_{i,9}) \cdot k_{i,10})} [B^{k_{i,12}}]_2 : 1 \leq i \leq 7, j = k_{i,4}.$$

Those which growth population depend on the fertility ratio and are retired of the ecosystem:

$$\blacktriangleright r_{15} \equiv [Y_{ij} \xrightarrow{(1-k_{i,15}) \cdot (1-k_{i,9}) \cdot (1-k_{i,10})} \lambda]_1 : 1 \leq i \leq 7, j = k_{i,4}.$$

Those which growth population not depend on the fertility ratio:

$$\blacktriangleright r_{16} \equiv Y_{ij}[ ]_2 \xrightarrow{k_{i,15}} [Z_{ik_{i,2}}]_2 : 1 \leq i \leq 7, j = k_{i,4}.$$



## Reglas de Alimentación

- ▶  $r_{17} \equiv [Z_{ij}B^{k_i,14}]_2 \rightarrow X_{ij+1}[ ]_2^+ : 1 \leq i \leq 7, 0 \leq j \leq k_{i,4}$ .

## Reglas de hacer balance al final del ciclo

Elimination of remaining bones:

- ▶  $r_{18} \equiv [B]_2^+ \rightarrow [ ]_2$ .

Adult animals that die because they have not enough food:

- ▶  $r_{19} \equiv [Z_{ij}]_2^+ \rightarrow [B^{k_i,12}]_2 : 1 \leq i \leq 7, k_{i,1} \leq j \leq k_{i,4}$

Young animals that die because they have not enough food:

- ▶  $r_{20} \equiv [Z_{ij}]_2^+ \rightarrow [B^{k_i,11}]_2 : 1 \leq i \leq 7, j < k_{i,1}$

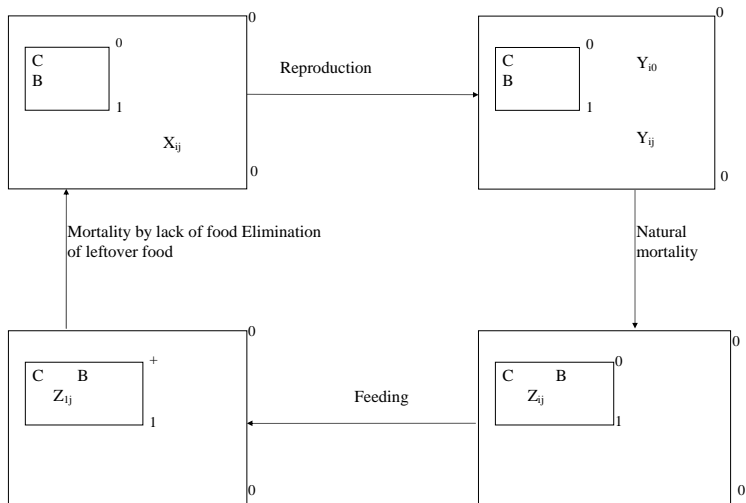
If the food is equal to the necessary object C allow to change the polarization:

- ▶  $r_{21} \equiv [C]_2^+ \rightarrow [C]_2$ .

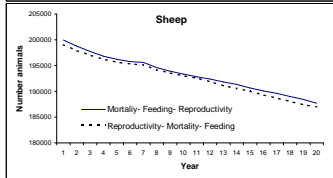
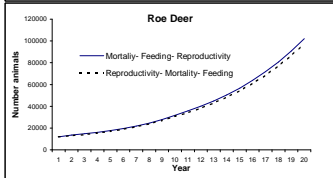
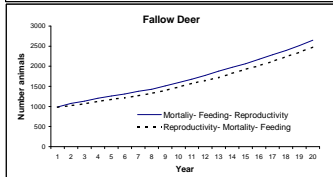
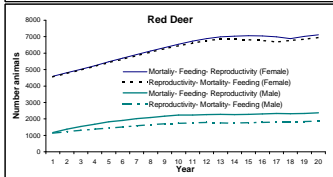
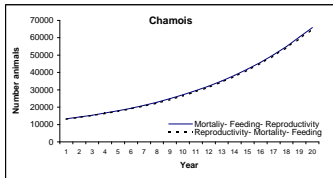
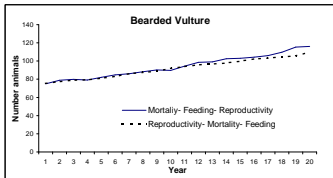
## Constantes asociadas a las reglas

- ▶  $k_{i,1}$ : age at which adult size is reached.
- ▶  $k_{i,2}$ : age at which it starts to be fertile.
- ▶  $k_{i,3}$ : age at which it stops being fertile.
- ▶  $k_{i,4}$ : average life expectancy in the ecosystem.
- ▶  $k_{i,5}$ : fertility ratio (number of descendants by fertile female).
- ▶  $k_{i,6}$ : population growth (this quantity is expressed in terms of 1).
- ▶  $k_{i,7}$ : Animals retired from the ecosystem in the first years ( $age < k_{i,1}$ ): this quantity is expressed in terms of 1.
- ▶  $k_{i,8}$ : mortality ratio in first years ( $age < k_{i,1}$ ): this quantity is expressed in terms of 1.
- ▶  $k_{i,9}$ : 0 if the live animals are retired at age  $k_{i,4}$ , 1 otherwise.
- ▶  $k_{i,10}$ : mortality ratio in adult animals ( $age \geq k_{i,1}$ ): this quantity is expressed in terms of 1.
- ▶  $k_{i,11}$ : Amount of bones from young animals,  $age < k_{i,1}$ .
- ▶  $k_{i,12}$ : Amount of bones from adult animals,  $edad \geq k_{i,1}$ .
- ▶  $k_{i,13}$ : Proportion of females in the population (this quantity is expressed in terms of 1).
- ▶  $k_{i,14}$ : Amount of food necessary per year and breeding pair (1 unit is equal to 0'5 kgs of bones).
- ▶  $k_{i,15}$ : 0 if the species remain in the same territory throughout their lives, 1 otherwise.

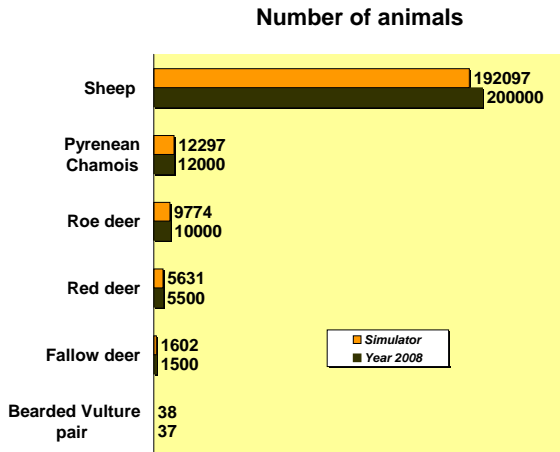
## Módulos del diseño



# Discusión y Resultados (I)

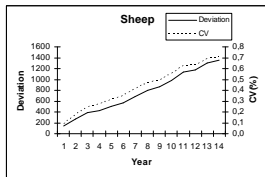
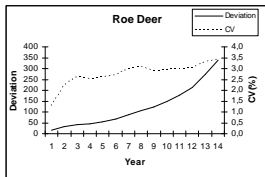
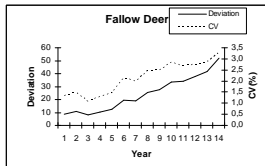
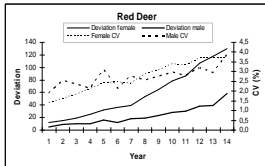
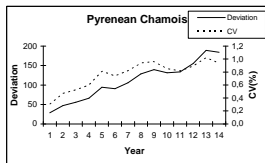
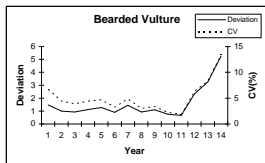


## Discusión y Resultados (II)



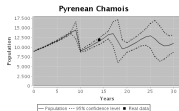
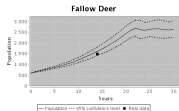
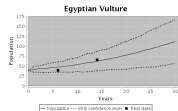
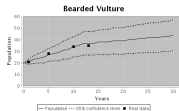
Average number of animals simulator vs census estimated in 2008

# Discusión y Resultados (III)

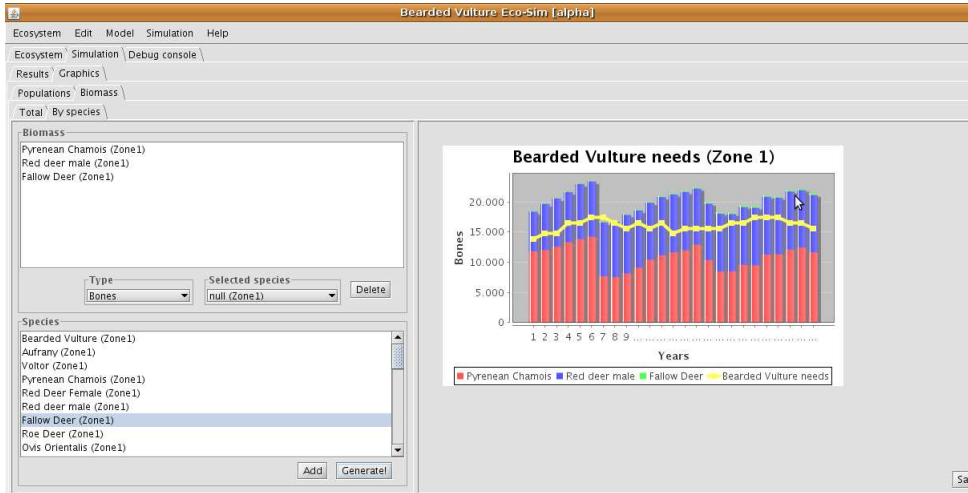


Deviation and Coefficient of Variation

# Discusiones y Resultados (IV)



# Discusiones y Resultados (V)



## SYSTEM USER

Data: /home/ignacio/Escritorio/TrencalosIII.ec2  
Model: /home/ignacio/Escritorio/TrencalosIIloc09.pli  
Simulated years: 25  
Simulations by year: 10  
Steps by year: 11

0%

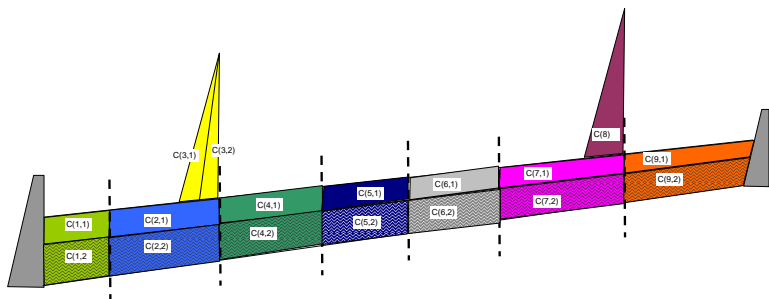
## Otro caso de estudio

### Ecosistema real del mejillón cebra<sup>3</sup>



- ▶ Especie exótica invasora.
- ▶ Pantano de Ribarroja, zona del Ebro (Endesa S.A.).

# El pantano de Ribarroja



# Un modelo del ecosistema del mejillón cebrá (I)

$$ME = (\Gamma, \Sigma, G, R_E, \Pi, \{M_{ij}\}_{1 \leq i \leq 17, 0 \leq j \leq 4})$$

where:

- ▶  $\Gamma = \{X_{sc}, Y_{sc}, U_{sc} : 1 \leq s \leq 6, 1 \leq c \leq 2\} \cup$
- ▶  $\Sigma = \{W_{0c}, W_{1c} : 1 \leq c \leq 2\}.$
- ▶  $\Pi = (\Gamma, \mu, R, \{c_r\}_{r \in R})$  is a skeleton polarization probabilistic P system of degree 5.
- ▶ The multisets of objects initially are:
  - ▶  $\{(M_m)_e\}_{1 \leq m \leq 3, 1 \leq e \leq 17} = \{X_{sc}\}_{1 \leq s \leq 6, 1 \leq c \leq 2}.$
  - ▶  $\{(M_4)_e\}_{1 \leq e \leq 17} = \{a, c_1\}.$
  - ▶  $\{(M_0)_e\}_{1 \leq e \leq 17} = \{\eta_0, T_1\}.$

# Un ecosistema real relacionado con el mejillón cebra (II)

$$\begin{aligned}\Gamma = & \{X_{s,c,a}, Y_{s,c,a} : 1 \leq s \leq 6, 1 \leq c \leq 2, 1 \leq a \leq T\} \cup \\ & \{W_{d,c,a}, Q_{d,c,a} : 0 \leq d \leq 189, 1 \leq c \leq 2, 1 \leq a \leq T\} \cup \\ & \{WE_{c,a,s} : 1 \leq c \leq 2, 1 \leq a \leq T, 1 \leq s \leq 17\} \cup \\ & \{L_{dc} : 0 \leq d \leq 28, 1 \leq c \leq 2\} \cup \{t_i, t'_i : 1 \leq i \leq 3\} \cup \\ & \{D_{cdf} : 1 \leq c \leq 2, 1 \leq d \leq 14, 1 \leq f \leq 7\} \cup \{D_c : 1 \leq c \leq 2\} \cup \\ & \{a_s : 0 \leq s \leq 5\} \cup \{c_i : 1 \leq i \leq 2\} \cup \{e_i : 1 \leq i \leq 6\} \cup \\ & \{f_i : 1 \leq i \leq 5\} \cup \{t, b, M, e, f\} \cup \{\eta_i : 0 \leq i \leq 153\}\end{aligned}$$

# Reglas (I)

$$r_1 \equiv [\eta_i]_0^0 \rightarrow [\eta_{i+1}]_0^0, \quad 0 \leq i \leq 151, i \neq 103, i \neq 136$$

$$r_2 \equiv [D_c \xrightarrow{f_{r_2,j}} \lambda]_0^0, \quad \left\{ \begin{array}{l} 1 \leq c \leq 2, \\ 1 \leq j \leq 17 \end{array} \right.$$

$$r_3 \equiv [D_c \xrightarrow{1-f_{r_2,j}} D_{c,1,1}]_0^0, \quad \left\{ \begin{array}{l} 1 \leq c \leq 2, \\ 1 \leq j \leq 17 \end{array} \right.$$

$$r_4 \equiv [D_{c,d,f} \xrightarrow{f_{r_4,j}} t_1, t_2, t_3]_0^0, \quad \left\{ \begin{array}{l} 1 \leq d \leq 14, \\ 1 \leq f \leq 7, \\ 1 \leq c \leq 2, \\ 1 \leq j \leq 17 \end{array} \right.$$

$$r_5 \equiv [D_{c,d,f} \xrightarrow{f_{r_5,j}} t_1, t_2, D_{c,d+1,f}]_0^0, \quad \left\{ \begin{array}{l} 1 \leq d \leq 14, \\ 1 \leq f \leq 7, \\ 1 \leq c \leq 2, \\ 1 \leq j \leq 17 \end{array} \right.$$

$$r_6 \equiv [D_{c,d,f} \xrightarrow{f_{r_6,j}} t_1, D_{c,d+1,f}]_0^0, \quad \left\{ \begin{array}{l} 1 \leq d \leq 14, \\ 1 \leq f \leq 7, \\ 1 \leq c \leq 2, \\ 1 \leq j \leq 17 \end{array} \right.$$

$$r_7 \equiv [D_{c,d,f} \xrightarrow{1-\sum_{i=4}^{i=7} f_{r_{icf}(7-i),j}} D_{c,d+1,f}]_0^0, \quad \left\{ \begin{array}{l} 1 \leq d \leq 14, \\ 1 \leq f \leq 7, \\ 1 \leq c \leq 2, \\ 1 \leq j \leq 17 \end{array} \right.$$

$$r_8 \equiv [D_{c,14,f} \xrightarrow{f_{r_8,j}} t_1, t_2, t_3]_0^0, \quad \left\{ \begin{array}{l} 1 \leq f \leq 7, \\ 1 \leq c \leq 2, \\ 1 \leq j \leq 17 \end{array} \right.$$

$$r_9 \equiv [D_{c,14,f} \xrightarrow{f_{r_9,j}} t_1, t_2, D_{c1f+1}]_0^0, \quad \left\{ \begin{array}{l} 1 \leq f \leq 7, \\ 1 \leq c \leq 2, \\ 1 \leq j \leq 17 \end{array} \right.$$

## Reglas (II)

$$r_{10} \equiv [D_{c,14,f} \xrightarrow{f_{r_{10},j}} t_1, D_{c,1,f+1}]_0^0, \begin{cases} 1 \leq f \leq 7, \\ 1 \leq c \leq 2, \\ 1 \leq j \leq 17 \end{cases}$$

$$r_{11} \equiv [D_{c,14,f} \xrightarrow{1 - \sum_{i=8}^{i=10} f_{r_{icf}(11-i),j}} D_{c,1,f+1}]_0^0, \begin{cases} 1 \leq f \leq 7, \\ 1 \leq c \leq 2, \\ 1 \leq j \leq 17 \end{cases}$$

$$r_{12} \equiv t_m [ ]_m^0 \rightarrow [t]_m^+, \quad 1 \leq m \leq 3$$

$$r_{13} \equiv [t]_m^+ \rightarrow t [ ]_m^0, \quad 1 \leq m \leq 3$$

$$r_{14} \equiv [Q_{d,c,a}]_m^+ \xrightarrow{0.5} Y_{1,c,a} t_{0,c,a}^{g_{c,1}} [ ]_m^0, \begin{cases} 1 \leq a \leq T, \\ n(j) \leq d \leq 180, \\ 1 \leq c \leq 2, \\ 1 \leq m \leq 3, \\ 1 \leq j \leq 17 \end{cases}$$

$$r_{15} \equiv [Q_{d,c,a}]_m^+ \xrightarrow{0.5} Y_{1,c,a} [ ]_m^0, \begin{cases} 1 \leq a \leq T, \\ n(j) \leq d \leq 180, \\ 1 \leq c \leq 2, \\ 1 \leq m \leq 3, \\ 1 \leq j \leq 17 \end{cases}$$

$$r_{16} \equiv [Q_{d,c,a}]_m^+ \rightarrow Y_{1,c,a} [ ]_m^0, \begin{cases} 1 \leq a \leq T, \\ 0 \leq d < n(j), \\ 1 \leq c \leq 2, \\ 1 \leq m \leq 3, \\ 1 \leq j \leq 17 \end{cases}$$

$$r_{17} \equiv [X_{s,c,a}]_m^+ \xrightarrow{0.5} Y_{s,c,a} t_{0,c,a}^{g_{c,s}} [ ]_m^0, \begin{cases} 1 \leq a \leq T, \\ 1 \leq s \leq 6, \\ 1 \leq c \leq 2, \\ 1 \leq m \leq 3, \end{cases}$$

$$r_{18} \equiv [X_{s,c,a}]_m^+ \xrightarrow{0.5} Y_{s,c,a} [ ]_m^0, \begin{cases} 1 \leq a \leq T, \\ 1 \leq s \leq 6, \\ 1 \leq c \leq 2, \\ 1 \leq m \leq 3, \end{cases}$$

# Reglas (III)

$$r_{19} \equiv [L_{d,c,a} \rightarrow L_{d+1,c,a}]_0^0, \begin{cases} 0 \leq d < 28, \\ 1 \leq a \leq T, \\ 1 \leq c \leq 2 \end{cases}$$

$$r_{20} \equiv [L_{28,c,a}]_0^0 \xrightarrow{f_{r_{20},j}} WE_{c,a,s}[ ]_0^0, \begin{cases} 1 \leq a \leq T, \\ 1 \leq c \leq 2, \\ 1 \leq s \leq 17, \\ 1 \leq j \leq 17 \end{cases}$$

$$r_{21} \equiv [L_{28,c,a} \xrightarrow{f_{r_{21},j}} \lambda]_0^0, \begin{cases} 1 \leq a \leq T, \\ 1 \leq c \leq 2, \\ 1 \leq j \leq 17 \end{cases}$$

$$r_{22} \equiv (WE_{c,a,s})_{e_j} \rightarrow (W_{1,c,a})_{e_s}, \begin{cases} 1 \leq c \leq 2, \\ 1 \leq a \leq T, \\ (e_j, e_s) \in S, \\ j \neq s \end{cases}$$

$$r_{23} \equiv (WE_{c,a,j})_{e_j} \rightarrow (W_{1,c,a})_{e_j}, \begin{cases} 1 \leq c \leq 2, \\ 1 \leq a \leq T, \\ 1 \leq j \leq 17 \end{cases}$$

$$r_{24} \equiv W_{1,c,a}[ ]_0^0 \rightarrow [W_{2,c,a}]_0^0, \begin{cases} 1 \leq a \leq T, \\ 1 \leq c \leq 2 \end{cases}$$

$$r_{25} \equiv [Y_{6,c,a} \rightarrow M]_0^0, \begin{cases} 1 \leq a \leq T, \\ 1 \leq c \leq 2 \end{cases}$$

$$r_{26} \equiv Y_{s,c,a}[ ]_4^0 \rightarrow [Y_{s,c,a}]_4^0, \begin{cases} 1 \leq s \leq 5, \\ 1 \leq a \leq T, \\ 1 \leq c \leq 2 \end{cases}$$

$$r_{27} \equiv W_{2,c,a}[ ]_4^0 \rightarrow [Q_{3,c,a}]_4^0, \begin{cases} 1 \leq a \leq T, \\ 1 \leq c \leq 2 \end{cases}$$

# Reglas (IV)

$$r_{28} \equiv [Q_{d,c,a} \rightarrow Q_{d+1,c,a}]_4^0, \left\{ \begin{array}{l} 3 \leq d \leq 180, \\ 1 \leq a \leq T, \\ 1 \leq c \leq 2 \end{array} \right.$$

$$r_{29} \equiv [\eta_{103} \rightarrow t'_1 t'_2 t'_3 \eta_{104}]_0^0$$

$$r_{30} \equiv [t'_m]_m^0 \rightarrow [t]_m^-, 1 \leq m \leq 3$$

$$r_{31} \equiv [X_{s,c,a}]_m^- \rightarrow Y_{s,c,a}[ ]_m^0, \left\{ \begin{array}{l} 1 \leq s \leq 6, \\ 1 \leq a \leq T, \\ 1 \leq c \leq 2, \\ 1 \leq m \leq 3 \end{array} \right.$$

$$r_{32} \equiv [Q_{d,c,a}]_m^- \rightarrow Y_{1,c,a}[ ]_m^0, \left\{ \begin{array}{l} 1 \leq d \leq 180, \\ 1 \leq a \leq T, \\ 1 \leq c \leq 2, \\ 1 \leq m \leq 3 \end{array} \right.$$

$$r_{33} \equiv [t]_m^- \rightarrow [t]_m^0, 1 \leq m \leq 3$$

$$r_{34} \equiv \eta_{138}[ ]_4^0 \rightarrow \eta_{139}[b]_4^+$$

$$r_{35} \equiv [h]_4^+ \rightarrow b[a_0^{B(k) \times 250000}]_4^0, 1 \leq k \leq 17$$

$$r_{36} \equiv [a_s]_4^+ \rightarrow b[a_{s+1}]_4^0, 0 \leq s \leq 4$$

$$r_{37} \equiv [e]_4^+ \rightarrow b[f_1]_4^0$$

$$r_{38} \equiv [e_i]_4^+ \rightarrow b[f_i]_4^0, 1 \leq i \leq 5$$

$$r_{39} \equiv [f_i]_4^0 \rightarrow b[e_{i+1}]_4^+, 1 \leq i \leq 5$$

$$r_{40} \equiv [e_6]_4^+ \rightarrow b[f]_4^0$$

$$r_{41} \equiv [Q_{d,c,a} a_0]_4^0 \rightarrow Q_{d+1,c,a}[ ]_4^+, \left\{ \begin{array}{l} 1 \leq a \leq T, \\ 3 \leq d \leq 180, \\ 1 \leq c \leq 2 \end{array} \right.$$

$$r_{42} \equiv [Y_{s,c,a} a_s]_4^0 \rightarrow X_{s+1,c+(-1)^{c+1},a}[ ]_4^+, \left\{ \begin{array}{l} 1 \leq a \leq T, \\ 1 \leq s \leq 5, \\ 1 \leq c \leq 2 \end{array} \right.$$

$$r_{43} \equiv X_{s,c,a}[ ]_m^0 \xrightarrow{f_{43,j}} [X_{s,c,a}]_m^0, \left\{ \begin{array}{l} 1 \leq a \leq T, \\ 1 \leq m \leq 3, \\ 1 \leq c \leq 2, \\ 1 \leq j < 17 \end{array} \right.$$

# Reglas (V)

$$r_{44} \equiv Q_{d,c,a} [ ]_m^0 \xrightarrow{f_{r_{44},j}} [Q_{d+1+(c-1)*60,c+(-1)^{c+1},a+(c-1)}]_m^0,$$

$$\left\{ \begin{array}{l} 1 \leq a \leq T, \\ 0 \leq d \leq 121, \\ 1 \leq m \leq 3, \\ 1 \leq c \leq 2, \\ 1 \leq j \leq 17 \end{array} \right.$$

$$r_{45} \equiv [Q_{d,c,a} \rightarrow Q_{d+1,c,a}]_m^0, \left\{ \begin{array}{l} 1 \leq a \leq T, \\ 5 \leq d \leq 121, \\ 1 \leq m \leq 3, \\ 1 \leq c \leq 2 \end{array} \right.$$

$$r_{46} \equiv \eta_{152} [ ]_4^0 \rightarrow \eta_{153} [h]_4^-$$

$$r_{47} \equiv [Y_{s,c,a}]_4^- \rightarrow M [ ]_4^0, \left\{ \begin{array}{l} 1 \leq s \leq 5, \\ 1 \leq c \leq 2, \\ 1 \leq a \leq T \end{array} \right.$$

$$r_{48} \equiv [Q_{d,c,a}]_4^- \rightarrow M [ ]_4^0, \left\{ \begin{array}{l} 1 \leq d \leq 121, \\ 1 \leq c \leq 2, \\ 1 \leq a \leq T \end{array} \right.$$

$$r_{49} \equiv [a_s]_4^- \rightarrow b [ ]_4^0, 0 \leq s \leq 6$$

$$r_{50} \equiv [b]_4^- \rightarrow b [ ]_4^0$$

$$r_{51} \equiv [b \rightarrow \lambda]_0^0$$

$$r_{52} \equiv [t \rightarrow \lambda]_0^0$$

$$r_{53} \equiv [\eta_{153} \rightarrow \eta_0]_0^0$$

$$r_{54} \equiv [c_i]_4^- \rightarrow qD_{i+(-1)^{i+1}} [c_{i+(-1)^{i+1}}]_4^0, 1 \leq i \leq 2$$

$$r_{55} \equiv [f]_4^- \rightarrow b[e]_0^4$$