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Păun's Conjecture Beyond Polarizations: Alternative Formulations

Carmen Graciani-Díaz, Luis F. Macías-Ramos,
Miguel A. Martínez-del-Amor, Mario J. Pérez-Jiménez,
Agustín Riscos-Núñez, Álvaro Romero-Jiménez,
Luis Valencia-Cabrera

Abstract - P systems with active membranes are models of membrane computing which use (three) electrical charges and rules of the form: evolution, communication (send-in and send-out), dissolution and division for elementary membranes. In that model there is no cooperation and labels of membranes remain unchanged. At the beginning of 2005, Gh. Păun set the (still open) problem to determine the role of the polarization from a computational complexity point of view, asking whether in the previous framework electrical charges can be completely avoided, while still being able to solve hard problems efficiently. His guess was that the answer is negative, and thus the polarizations would provide a boundary between tractability and **NP**–hardness.

In this paper a brief survey of results about Păun's conjecture is provided, and several further topics are considered. Specifically, new formulations of the conjecture are described, highlighting the role of some other syntactical ingredients from a computational complexity point of view.

Key words and phrases : Polarizationless P systems with active membranes, Păun's conjecture, Dependency graph.

1 Introduction

Membrane Computing is a discipline of *Natural Computing* initiated by Gh. Păun at the end of 1998, aiming to abstract computing models from the compartmentalized structure and the functioning of biological membranes within living cells, as well as from the way in which cells are organized in tissues, organs, colonies or higher order structures. It has received important attention from the scientific community since then, with contributions by computer scientists, formal linguists, biologists, and computational complexity theoreticians, enriching each others with results, open problems and promising new research lines. In fact, membrane computing was selected by the Institute for Scientific Information, USA, as a fast *Emerging Research Front* in computer science, and the seminal paper was mentioned as a highly

cited paper in October 2003. The devices of this computing paradigm, called *P systems* or *membrane systems*, constitute models for distributed, parallel and non-deterministic computing.

In the seminal paper [11], a rule-based computing mechanism essentially designed as a distributed parallel machinery, called *transition P systems*, was introduced. Roughly speaking, a transition P system consists of a cell-like (compartmentalized) membrane structure (formally, a rooted tree whose nodes are called membranes, the root is called the skin membrane and the leaves are called elementary membranes), in the compartments of which one places multisets of objects which evolve according to given evolution rules in a synchronous, non-deterministic, maximally parallel manner. In these P systems, the number of membranes in the initial membrane structure can decrease in time, but never increase. Nevertheless, in these systems an exponential workspace, in terms of the number of objects, can be constructed in linear time, for instance by considering rules that duplicate some objects in a computation step.

With respect to the capability of transition P systems to solve decision problems we stress two first results. On one hand, every deterministic Turing machine working in polynomial time can be simulated in polynomial time by a family of transition P systems [21]. On the other hand, each *confluent* transition P system (all possible computations of the system must give the same answer) can be efficiently simulated by a deterministic Turing machine [7]. So, if a decision problem is solvable in polynomial time by a family of transition P systems, then there exists a deterministic Turing machine solving it in polynomial time. As a consequence of these results, the class of all decision problems solvable in polynomial time by this kind of P systems is equal to the standard complexity class **P** [7]. For that reason, assuming that $\mathbf{P} \neq \mathbf{NP}$, transition P systems cannot efficiently solve **NP**–complete problems. Thus, the ability of these P systems to construct an exponential workspace (in terms of number of objects) in linear time is not enough to efficiently solve computationally hard problems. Therefore, new ingredients are necessary in order to be able to provide efficient solutions of computationally hard problems by making use of an exponential workspace, expressed in terms of number of membranes and number of objects, created in linear time.

Cell division is one of the basic processes in the cell life cycle and it allows producing two or more cells from one cell that could be considered the "mother". Basically, there are three processes associated to cell division: *binary fission* (typical for prokaryotic cells), *mitosis* and *meiosis* (these two taking place in eukaryotic cells). Several cell-division-inspired mechanisms were introduced in membrane computing through *cell division* rules and *membrane division* rules that are triggered by an object which is replaced in the two new cells by possibly new objects and the remaining objects are

duplicated both in the created cells/membranes. These two ways have given rise to cell-like P systems (*with active membranes* [12]) and tissue-like P systems (*with cell division* [15]).

P systems with active membranes [13] incorporate division rules (for elementary or non-elementary membranes) that are an abstraction of *mitosis*. In these systems, membranes have associated electrical charges from the set $\{0, +, -\}$, with 0 being the neutral polarization, and their computational completeness (equivalent in power to deterministic Turing machines) and computational efficiency (ability so solve hard problems in polynomial time) have been established. It would be interesting to analyze which syntactic ingredients are essential, as far as efficiency is concerned. In this context, at the beginning of 2005, Gh. Păun asks if electrical charges can be completely avoided while still being able to efficiently solve hard problems by means of P systems with active membranes. Moreover, he guesses that the answer is negative, in the sense that polarizations cannot be completely avoided, so the so-called *Păun's conjecture* is expressed in terms of electrical charges. In this paper, we study different aspects related to that guess: partial solutions, a reformulation in terms of division rules for non–elementary membranes and some relationships with the **P** versus **NP** problem.

The paper is structured as follows. Next, some concepts, results and the specific membrane computing framework that we use in the work are briefly described. In Section 3, Păun's conjecture is recalled, and some partial answers are considered by using the dependency graph technique. Section 4 is devoted to provide new formulations of the conjecture, and some consequences are briefly studied assuming that the conjecture holds. We conclude with some brief final remarks.

2 Preliminaries

The present work is developed within the framework of P systems with active membranes. In order to make this paper self-contained, let us introduce next some preliminary concepts which will be used in the sequel.

2.1 Languages and Multisets

An *alphabet* Γ is a non-empty set whose elements are called *symbols*. A *string* u over Γ is a mapping from a natural number $n \in \mathbb{N}$ onto Γ, that is, a finite sequence over Γ. A *language* over Γ is a set of strings over Γ.

A *multiset* over an alphabet Γ is an ordered pair (Γ, f), where f is a mapping from Γ onto the set of natural numbers $\mathbb{N}$. The *support* of a multiset $m = (\Gamma, f)$ is defined as $supp(m) = \{x \in \Gamma \mid f(x) > 0\}$. A multiset is finite if its support is a finite set. We denote by $M_f(\Gamma)$ the set of all finite multisets over Γ.

Let $m_1 = (\Gamma, f_1)$, $m_2 = (\Gamma, f_2)$ be multisets over Γ, then the union of m_1 and m_2, denoted by $m_1 + m_2$, is the multiset (Γ, g), where $g(x) = f_1(x) + f_2(x)$ for each $x \in \Gamma$. The relative complement of m_2 in m_1, denoted by $m_1 \backslash m_2$, is the multiset (Γ, g), where $g(x) = f_1(x) - f_2(x)$ if $f_1(x) \geq f_2(x)$, and $g(x) = 0$ otherwise.

2.2 The Graph Reachability Problem and the Circuit Value Problem

A *rooted tree* is a connected, acyclic, undirected graph in which one of the vertices (called *the root of the tree*) is distinguished from the others. Given a node x (different from the root) in a rooted tree, if the last edge on the (unique) path from the root to node x is $\{x, y\}$ (so $x \neq y$), then y is **the** *parent* of node x and x is **a** *child* of node y. We denote it by $y = p(x)$ and $x \in ch(y)$. The root is the only node in the tree with no parent. A node with no children is called a *leaf* (see [4] for details).

A *decision problem*, X, is a pair (I_X, θ_X) such that I_X is a language over a finite alphabet (whose elements are called *instances*) and θ_X is a total Boolean function (that is, a predicate) over I_X.

The `REACHABILITY` (or accessibility) problem is the following decision problem: *given a directed graph $G = (V, E)$ with two specified vertices s and t, determine whether or not there is a path from s to t.*

There are algorithms solving this problem, for instance, search algorithms based on breadth-first search or depth-first search. These algorithms determine whether two vertices are connected in $O(\max(|V|, |E|))$ time. Moreover, they basically need to store at most $|V|$ items, so these algorithms use $O(|V|)$ space. But this quantity of space can be reduced to $O(\log^2|V|)$ by using an algorithm that could be called middle-first search (see [10] for details, pp. 149-150). In particular, `REACHABILITY` $\in \mathbf{P}$.

The `CIRCUIT VALUE` problem is the following decision problem: *given a circuit with no variable gates and where each input gate has an associated Boolean value, determine whether or not the circuit evaluates to* `true`.

It is well known that the circuit value problem is a **P**–complete problem (see theorem 8.1 in [10] for details).

2.3 P system with active membranes

Let us recall some basic definitions of P systems with active membranes (see [13, 16] for details).

Definition 2.1 *A P system with active membranes of degree $q \geq 1$ is a tuple $\Pi = (\Gamma, H, \mu, \mathcal{M}_1, \ldots, \mathcal{M}_q, \mathcal{R})$, where:*

1. *Γ is a finite alphabet whose elements are called objects;*

2. *H is a finite set of labels for membranes;*

3. *μ is a membrane structure (a rooted tree) of q membranes, injectively labelled with elements of H, and having neutral polarization;*

4. *$\mathcal{M}_1, \ldots, \mathcal{M}_q$ are finite multisets over Γ, describing the initial objects placed in the q regions of μ (each membrane delimits a region bounded by a membrane and the immediately lower membranes, if any);*

5. *$\mathcal{R}$ is a finite set of evolution rules, of the following forms:*

   (a) $[\, a \to u\,]_h^\alpha$, *for* $h \in H$, $\alpha \in \{+, -, 0\}$, $a \in \Gamma$, $u \in M_f(\Gamma)$ (*object evolution rules*);

   (b) $a\,[\ \ ]_h^{\alpha_1} \to [\, b\,]_h^{\alpha_2}$, *for* $h \in H$, $\alpha_1, \alpha_2 \in \{+, -, 0\}$, $a, b \in \Gamma$ (*send-in rules*);

   (c) $[\, a\,]_h^{\alpha_1} \to b\,[\ \ ]_h^{\alpha_2}$, *for* $h \in H$, $\alpha_1, \alpha_2 \in \{+, -, 0\}$, $a, b \in \Gamma$ (*send-out rules*);

   (d) $[\, a\,]_h^\alpha \to b$, *for* $h \in H$, $\alpha \in \{+, -, 0\}$, $a, b \in \Gamma$ (*dissolution rules*);

   (e) $[\, a\,]_h^{\alpha_1} \to [\, b\,]_h^{\alpha_2}\, [\, c\,]_h^{\alpha_3}$, *for* $h \in H$, $\alpha_1, \alpha_2, \alpha_3 \in \{+, -, 0\}$, $a, b, c \in \Gamma$ (*division rules for elementary membranes*);

   (f) $[\,[\,]_{h_1}^{\alpha_1} \ldots [\,]_{h_k}^{\alpha_1}\ [\,]_{h_{k+1}}^{\alpha_2} \ldots [\,]_{h_n}^{\alpha_2}\,]_h^\alpha \to [\,[\,]_{h_1}^{\alpha_3} \ldots [\,]_{h_k}^{\alpha_3}\,]_h^\beta\, [\,[\,]_{h_{k+1}}^{\alpha_4} \ldots [\,]_{h_n}^{\alpha_4}\,]_h^\gamma$, *for* $n > k \geq 1$, $h, h_1, \ldots, h_n \in H$, $\alpha, \beta, \gamma, \alpha_1, \ldots, \alpha_4 \in \{+, -, 0\}$ $\{\alpha_1, \alpha_2\} = \{+, -\}$ (*division rules for non–elementary membranes*).

These rules are applied in a non-deterministic maximally parallel manner, according to some principles (see [13] for details). Note that these P systems have some important features: (a) They use three electrical charges; (b) The polarization of a membrane can be modified by the application of a rule; (c) The label of a membrane cannot be modified by the application of a rule; (d) They do not use cooperation, neither priorities.

P systems can be used as recognizer devices. In that case, the working alphabet Γ has two distinguished elements `yes` and `no`, there is an input alphabet Σ strictly contained in the working alphabet, the initial multisets associated with membranes are multisets over $\Gamma \setminus \Sigma$, all computations halt, and either object `yes` or object `no` (but not both) is sent out to the environment, and only in the last computation step. These systems have an input membrane where instances of problems are supplied to them by using a suitable encoding. For each finite multiset w over Σ, the *initial configuration* of Π with *input multiset* w is obtained by adding multiset w to the contents of the input membrane i_{in} (we denote it by $\Pi + w$).

The class of recognizer P systems with active membranes using division rules (which do not make use of division rules, resp.) is denoted by $\mathcal{AM}$ ($\mathcal{NAM}$, resp.). Similarly, $\mathcal{AM}(\alpha, \beta)$, where $\alpha \in \{-d, +d\}$, $\beta \in \{-ne, +ne\}$, denotes the class of all recognizer P systems with active membranes such

that: (a) if $\alpha = +d$ ($\alpha = -d$, resp.) then dissolution rules are permitted (forbidden, resp.); and (b) if $\beta = +ne$ ($\beta = -ne$, resp.) then division rules for non–elementary membranes are permitted (forbidden, resp.).

Next, let us recall the concept of efficient solvability by means of a family of membrane systems (see [17] for more details).

Definition 2.2 *A decision problem X is solvable in polynomial time by a family $\mathbf{\Pi} = \{\Pi(n) \mid n \in \mathbb{N}\}$ of recognizer P systems with active membranes, in a uniform way, denoted by $X \in \mathbf{PMC}_{\mathcal{R}}$, if the following hold:*

1. *The family $\mathbf{\Pi}$ is* polynomially uniform *by Turing machines.*
2. *There exists a pair (cod, s) of polynomial-time computable functions over the set of instances of X such that: (a) for each instance u, $s(u)$ is a natural number and $cod(u)$ is an input multiset of $\Pi(s(u))$; (b) for each $n \in \mathbb{N}$, $s^{-1}(n)$ is a finite set; and (c) the family $\mathbf{\Pi}$ is* polynomially bounded*,* sound *and* complete *with respect to (X, cod, s).*

The polynomial complexity class $\mathbf{PMC}_{\mathcal{R}}$ is closed under polynomial-time reduction and under complement [22].

P systems with active membranes (without dissolution and using division rules only for elementary membranes) have been successfully used to design polynomial time solutions to **NP**–complete problems (e.g. `SAT` [22], `Subset Sum` [19], `Knapsack` [18], `Partition` [5], etc.). It is important to note that some of these solutions only make use of two polarizations in their design.

On the other hand, from the proof of the Milano theorem [26] (*each deterministic P system with active membranes but without membrane division can be simulated by a deterministic Turing machine with a polynomial slowdown*) and from a proof given by A.E. Porreca [23] (*each tractable problem can be solved in polynomial time by families of recognizer P systems with active membranes and without input*), we have $\mathbf{PMC}_{\mathcal{NAM}} = \mathbf{P}$. Therefore, in the framework of P systems with active membranes and electrical charges, division rules (only for elementary membranes) provide a borderline between tractability and intractability (assuming that $\mathbf{P} \neq \mathbf{NP}$).

Nevertheless, P systems with active membranes seem to be too powerful from a complexity point of view. On one hand, the polynomial complexity class associated with P systems with elementary active membranes is contained in **PSPACE**. On the other hand, that class of P systems can solve efficiently all $\mathbf{P}^{\#\mathbf{P}}$ problems [24], that is all decision problems recognizable in polynomial time by deterministic Turing machines with oracles for $\#\mathbf{P}$ functions (see [10] for details). Moreover, recently it has been shown that complexity class $\mathbf{PMC}_{\mathcal{AM}(+d,+ne)}$ is equal to the class $\mathbf{P}^{\#\mathbf{P}}$ [8]. Therefore, it would be interesting to remove some ingredients from P systems with active membranes in order to obtain new frontiers of the efficiency. Each of such boundaries will provide a tool to tackle the **P** versus **NP** problem.

Polarizationless P systems with active membranes are defined in a similar way to standard P systems with active membranes. In this case, the rules are of the following forms:

(a) $[\, a \to u \,]_h$, for $h \in H$, $a \in \Gamma$, $u \in M_f(\Gamma)$ *(object evolution rules)*;

(b) $a[\ \ ]_h \to [\, b \,]_h$, for $h \in H$, $a, b \in \Gamma$ *(send–in communication rules)*;

(c) $[\, a \,]_h \to [\ \ ]_h\, b$, for $h \in H$, $a, b \in \Gamma$ *(send–out communication rules)*;

(d) $[\, a \,]_h \to b$, for $h \in H$, $a, b \in \Gamma$ (*dissolution rules*);

(e) $[\, a \,]_h \to [\, b \,]_h\, [\, c \,]_h$, for $h \in H$, $a, b, c \in \Gamma$ (*division rules for elementary membranes*);

(f) $[\, [\,]_{h_1}\ [\,]_{h_2} \,]_h \to [\, [\,]_{h_1} \,]_h\, [\, [\,]_{h_2} \,]_h$, for $h, h_1, h_2 \in H$ (*division rules for non-elementary membranes).*

Classes of recognizer polarizationless P systems with active membranes are defined in a similar way: $\mathcal{AM}^0$, $\mathcal{NAM}^0$, and $\mathcal{AM}^0(\alpha, \beta)$, where $\alpha \in \{-d, +d\}$ and $\beta \in \{-ne, +ne\}$.

Let us note that class **P** is a subset of $\mathbf{PMC}_{\mathcal{AM}^0(-d,-ne)}$ because this class is closed under polynomial–time reduction and it contains the **P**–complete problem `CIRCUIT VALUE`.

The computational efficiency of polarizationless P systems with active membranes has been explored, and polynomial time (uniform) solutions for **NP**–complete problems [2, 6] and for **PSPACE**–complete problems [3] were given in this framework by using division for non-elementary membranes.

3 A conjecture of Păun

At the beginning of 2005, Gh. Păun (problem **F** from [14]) wrote:

> *My favorite question (related to complexity aspects in P systems with active membranes and with electrical charges) is that about the number of polarizations. Can the polarizations be completely avoided? The feeling is that this is not possible – and such a result would be rather sound: passing from no polarization to two polarizations amounts to passing from non–efficiency to efficiency.*

This so–called Păun's conjecture can be formally formulated in terms of membrane computing complexity classes as follows: $\mathbf{P} = \mathbf{PMC}_{\mathcal{AM}^0}$.

An *affirmative answer* to the conjecture would indicate that the ability to create an exponential amount of workspace (expressed in terms of the number of membranes and objects) in polynomial time, is not enough in order to solve computationally hard problems efficiently. Conversely, a *negative*

answer to the conjecture would provide a borderline between tractability and intractability (assuming that $\mathbf{P} \neq \mathbf{NP}$): division rules for elementary membranes.

3.1 Limits on efficient computations in polarizationless P systems with active membranes without dissolution rules

Let Π be a recognizer polarizationless P system with active membranes which do not make use of dissolution rules. We try to *capture the dynamics* of such a P system by means of a static directed graph in the sense that the existence of an accepting computation of Π is characterized by the existence of a path in the graph between two specific nodes.

The rules of Π can be considered, in a certain sense, as a *dependency* between the object triggering the rule and the objects produced by its application.

- The rules of type $[\, a \to u \,]_h$ can be described as: the pair (a, h) produces the pair (b, h), for each $b \in supp(u)$.
- The rules of type $a[\ ]_h \to [\, b \,]_h$ can be described as: the pair $(a, p(h))$ produces the pair (b, h).
- The rules of type $[\, a \,]_h \to [\ ]_h\ b$ can be described as: the pair (a, h) produces the pair $(b, p(h))$.
- The rules of type $[\, a \,]_h \to [\, b \,]_h\, [\, c \,]_h$ can be described as: the pair (a, h) produces the pairs (b, h) and (c, h).

We formalize these ideas in the following definition.

Definition 3.1 *Let $\Pi = (\Gamma, H, \mu, \mathcal{M}_1, \ldots, \mathcal{M}_q, \mathcal{R})$ be a recognizer polarizationless P system from $\mathcal{AM}^0(-d, +ne)$. The dependency graph associated with Π is the directed graph $G_\Pi = (V_\Pi, E_\Pi)$ defined as follows:*

- *The set of vertices is $V_\Pi = \{s_\Pi\} \cup VL_\Pi \cup VR_\Pi$, where $s \notin \Gamma \cup H$ and:*

$$\begin{aligned} VL_\Pi = \{(a,h) \in \Gamma \times H \mid\ & \exists u \in M_f(\Gamma)\ ([a \to u]_h \in \mathcal{R}) \vee \\ & \exists b \in \Gamma\ ([a]_h \to [\ ]_h b \in \mathcal{R}) \vee \\ & \exists b \in \Gamma\ \exists h' \in ch(h)\ (a[\ ]_{h'} \to [b]_{h'} \in \mathcal{R}) \vee \\ & \exists b, c \in \Gamma\ ([a]_h \to [b]_h[c]_h \in \mathcal{R}))\}. \end{aligned}$$

$$\begin{aligned} VR_\Pi = \{(b,h) \in \Gamma \times H \mid\ & \\ & \exists a \in \Gamma\ \exists u \in M_f(\Gamma)\ ([a \to u]_h \in \mathcal{R} \wedge b \in supp(u)) \vee \\ & \exists a \in \Gamma\ \exists h' \in ch(h)\ ([a]_{h'} \to [\ ]_{h'} b \in \mathcal{R}) \vee \\ & \exists a \in \Gamma\ (a[\ ]_h \to [b]_h \in \mathcal{R}) \vee \\ & \exists a, c \in \Gamma\ ([a]_h \to [b]_h[c]_h \in \mathcal{R} \vee [a]_h \to [c]_h[b]_h \in \mathcal{R})\}. \end{aligned}$$

- *The set of arcs is* $E_\Pi = E^1_\Pi \cup E^2_\Pi$, *where:*

$$E^1_\Pi = \{(s_\Pi, (a,h)) \mid \textit{the object } a \textit{ is in membrane } h \textit{ at the initial configuration of } \Pi\}.$$

$$\begin{aligned} E^2_\Pi = \{((a,h),(b,h')) \mid & \\ & \exists u \in M_f(\Gamma)\ ([a \to u]_h \in \mathcal{R} \wedge b \in alph(u) \wedge h = h') \vee \\ & ([a]_h \to [\ ]_h b \in \mathcal{R}\ \wedge\ h' = f(h)) \vee \\ & (a[\ ]_{h'} \to [b]_{h'} \in \mathcal{R}\ \wedge h\ = f(h')) \vee \\ & \exists c \in \Gamma\ ([a]_h \to [b]_h[c]_h \in \mathcal{R} \wedge h = h') \vee \\ & \exists c \in \Gamma\ ([a]_h \to [c]_h[b]_h \in \mathcal{R} \wedge h = h')\}. \end{aligned}$$

Bearing in mind that all computations of a recognizer P system halt, we deduce that each path in the dependency graph associated with it must be simple.

3.2 Partial answers to Păun's conjecture

The dependency graph associated with a P system from $\mathcal{AM}^0(-d,+ne)$, can be constructed by a deterministic Turing machine working in polynomial time (see [6] for details). Moreover, dependency graphs can be used to characterize the behavior of the system through the analysis of simple paths. Specifically, given a recognizer P system Π from $\mathcal{AM}^0(-d,+ne)$, there exists an accepting computation of Π if and only if there exists a simple path in the dependency graph G_Π from s_Π to (yes, env) with length greater than or equal to 2 (see [6] for details).

Let $\{\Pi(n) \mid n \in \mathbb{N}\}$ be a family of P systems from $\mathcal{AM}^0(-d,+ne)$ solving a decision problem X in polynomial time. Let (cod, s) be the associated polynomial encoding. Then, for each instance w, the answer of the problem is `yes` if and only if there exists a simple path in the dependency graph associated with $\Pi' = \Pi(s(w)) + cod(w)$, from $s_{\Pi'}$ to (yes, env).

We thus can conclude that $\mathbf{PMC}_{\mathcal{AM}^0(-d,+ne)} = \mathbf{P}$. Indeed, in order to show that $\mathbf{PMC}_{\mathcal{AM}^0(-d,+ne)} \subseteq \mathbf{P}$, let $X \in \mathbf{PMC}_{\mathcal{AM}^0(-d,+ne)}$ and $\{\Pi(n) \mid n \in \mathbb{N}\}$ a family of P systems from $\mathcal{AM}^0(-d,+ne)$ solving X in a uniform way and in polynomial time. Let (cod, s) be a polynomial encoding associated with that solution. Let us see that there exists a polynomial time reduction from X to `REACHABILITY` problem. For that, let us consider the mapping F from I_X to $I_{\texttt{REACHABILITY}}$ defined as follows: $F(w) = (G_{\Pi(s(w))+cod(w)}, s_{\Pi(s(w))+cod(w)}, (yes, env))$, for each instance w of X. Then, F is a polynomial time computable function such that

$$w \in L_X \Longleftrightarrow \theta_X(w) = 1 \Longleftrightarrow F(w) \in L_{\texttt{REACHABILITY}}$$

Finally, we deduce that $X \in \mathbf{P}$ because the class $\mathbf{P}$ is closed under polynomial–time reductions, $X \leq^p$ `REACHABILITY` and `REACHABILITY` $\in \mathbf{P}$.

Theorem 3.1 $\mathbf{P} = \mathbf{PMC}_{\mathcal{AM}^0\,(-d,+ne)}$.

In particular, $\mathbf{P} = \mathbf{PMC}_{\mathcal{AM}^0\,(-d,-ne)}$. Hence, in the framework of polarizationless P systems with active membranes which do not make use of dissolution rules we have a *partial affirmative* answer to Păun's conjecture. The answer is partial because dissolution rules have been forbidden.

It is worth pointing out that N. Murphy and D. Woods [9] showed that polarizationless P systems with active membranes when division rules are used only for elementary membranes and being *symmetric*, in the following sense $[\,a\,]_h \rightarrow [\,b\,]_h[\,b\,]_h$, cannot efficiently solve hard problems, that is, $\mathbf{P} = \mathbf{PMC}_{\mathcal{AM}^0\,(+d,-n(sym))}$. This is also a partial affirmative solution to Păun's conjecture refered to a particular case of division for elementary membranes.

On the other hand, A. Alhazov and M.J. Pérez–Jiménez [3] have given a family of recognizer polarizationless P systems with active membranes using dissolution rules and division for non–elementary membranes solving `QBF-SAT` in a *uniform* way and in a linear time, that is, $\mathbf{PSPACE} \subseteq \mathbf{PMC}_{\mathcal{AM}^0\,(+d,+ne)}$.

In this manner, assuming that $\mathbf{P} \neq \mathbf{NP}$, we have a *partial negative* answer to Păun's conjecture: computationally hard problems can be efficiently solved avoiding polarizations. The answer is partial because division rules for non–elementary membranes have been considered.

From the previous result we deduce that when dealing with polarizationless P systems with active membranes including non–elementary division, dissolution rules are a key ingredient to solve hard problems efficiently.

4 Reformulations of Păun's conjecture

Păun's conjecture was initially formulated **in terms of polarizations**, that is, in order to solve hard problems, can polarizations be avoided? Gh. Păun's guess was a negative answer. Now, we show that the conjecture can be expressed in different terms.

(a) P. Sosík and A. Rodríguez-Patón [25] proved that the class of decision problems solvable in polynomial time by families of P systems with active membranes and with non-elementary membrane division is exactly the class **PSPACE**. Noting that $\mathbf{PSPACE} \subseteq \mathbf{PMC}_{\mathcal{AM}^0\,(+d,+ne)}$ and $\mathbf{PMC}_{\mathcal{AM}\,(+d,+ne)} = \mathbf{PSPACE}$ we deduce that $\mathbf{PMC}_{\mathcal{AM}^0\,(+d,+ne)} = \mathbf{PSPACE}$.

Then, Păun's conjecture can be reformulated as follows: in order to efficiently solve hard problems in the framework of polarizationless P systems with active membranes allowing dissolution rules, can **division for non–elementary membranes** be avoided?

(b) From Theorem 3.1 we deduce that $\mathbf{P} = \mathbf{PMC}_{\mathcal{AM}^0\,(-d,-ne)}$. Then, Păun's conjecture can be reformulated as follows: in order to efficiently solve hard problems in the framework of polarizationless P systems

with active membranes and division for elementary membranes, are **dissolution rules** relevant?

4.1 If Păun's conjecture holds ...

Let us assume that Păun's conjecture holds, that is, $\mathbf{P} = \mathbf{PMC}_{\mathcal{AM}^0\,(+d,-ne)}$.

(a) Recalling that $\mathbf{NP} \cup \mathbf{co} - \mathbf{NP} \subseteq \mathbf{PMC}_{\mathcal{AM}_{(+d,-ne)}}$ we deduce that in the framework of P systems with active membranes (allowing evolution, communication, dissolution and division for elementary membranes rules), passing from allowing electrical charges to forbidding them amounts to passing from efficiency to non–efficiency.

(b) To solve the **P** versus **NP** problem is equivalent to show if there exists an **NP**–complete problem X such that $X \in \mathbf{PMC}_{\mathcal{AM}^0_{(+d,-ne)}}$, in the following sense: if there exists such an **NP**–complete problem, then $\mathbf{P} = \mathbf{NP}$; otherwise, $\mathbf{P} \neq \mathbf{NP}$. Therefore, if there exists an **NP**–complete problem X such that $X \in \mathbf{PMC}_{\mathcal{AM}^0_{(+d,-ne)}}$ and $\mathbf{P} \neq \mathbf{NP}$, then Păun's conjecture will be false.

(c) Let us suppose that division rules for non–elementary membranes are irrelevant in order to efficiently solve hard problems in the framework of polarizationless P systems with active membranes, that is, let us suppose that $\mathbf{PMC}_{\mathcal{AM}^0_{(+d,+ne)}} \subseteq \mathbf{PMC}_{\mathcal{AM}^0_{(+d,-ne)}}$. Then we have $\mathbf{P} = \mathbf{PSPACE}$, in particular $\mathbf{P} = \mathbf{NP}$.

5 Conclusions

Păun's conjecture is still an open problem. In its original formulation, it refers to the role played by the polarizations in the framework of P systems with active membranes from a computational complexity point if view. Specifically, the question is whether electrical charges can be avoided when we try to solve **NP**–complete problems efficiently. A brief survey of related partial results has been provided in this paper, and new formulations of the conjecture are presented. For instance, it is well known that dissolution rules are not necessary for efficiently solving computationally hard problems, when working with P systems with active membranes [20]. However, as it was shown in the previous section, Păun's conjecture can be restated in terms of dissolution rules, and it is an open problem to determine if they play an essential role in the case of polarizationless P systems with active membranes.

Finally, it is worth noting that the relationships with the **P** versus **NP** problem described above stress the interest and relevance of the conjecture, not only for membrane computing, but for theoretical computer science in general.

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Authors
Carmen Graciani-Díaz, Luis F. Macías-Ramos, Miguel A. Martínez-del-Amor,
Mario J. Pérez-Jiménez, Agustín Riscos-Núñez,
Álvaro Romero-Jiménez, Luis Valencia-Cabrera
Research Group on Natural Computing
Department of Computer Science and Artificial Intelligence. Universidad de Sevilla
Avda. Reina Mercedes s/n, 41012 Sevilla, Spain
E-mail: {cgdiaz,lfmaciasr,mdelamor,marper,ariscosn,alvaro,lvalencia}@us.es