Computational Complexity Aspects of Membrane Computing: Ideas, Results, Open Problems

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Summary. In this paper, a brief survey of a theory of computational complexity within the model of cell-like membrane systems is presented. Relevant results concerning complexity classes in membrane computing are described, and fifteen open problems in this framework are proposed.

1 Introduction

The *theory of computation* deals with the *mechanical solvability* of problems and the most important question in this framework is basically qualitative: is a given decision problem *solvable*? However, some problems that are solvable in principle are not really solvable in practice, because their solutions would require huge amounts ot computational resources (time or space). Hence, from a practical point of view it is important to take real–life problems and try to solve them by a specific machine, hence in a framework where the computational resources available are limited.

The theory of *computational complexity* deals with the *solvability in practice*, studying the amount of computational resources needed to carry out computations, with the resources measured in general through the time or storage space used. There are problems solvable in principle for which even instances of a small size are too hard in practice. A classification of decision problems according to their inherent complexity is provided through the standard *complexity classes*.

The following parameters are used to specify a *complexity class*: (a) a *model* of computation, (b) a *mode* of computation, (c) a *resource* that we wish to bound, and (d) an upper *bound* of the resources.

Membrane Computing is a relatively young branch of Natural Computing initiated in the fall of 1998 by Gheorghe Păun, providing distributed parallel computing models whose computational devices are called *membrane systems*, or *P systems*. These systems are inspired by some basic biological features, by the structure and functioning of the living cells, as well as from the cooperation of cells in tissues, organs, and organisms.

In this area there are basically two ways to consider computational devices: cell–like membrane systems and tissue–like membrane systems. The first one uses the biological membranes arranged hierarchically, inspired from the structure of the cell, and the second one uses the biological membranes placed in the nodes of a graph, inspired from the cell inter–communication in tissues.

In this paper, we only work with cell-like membrane systems, called P systems. These computational devices are both able of Turing universal computations and able to solve computationally hard problems in a polynomial time, by trading space for time (quantified in an exponential number of membranes constructed in a polynomial time).

In this paper we present cell–like recognizer membrane systems as a framework to attack the solvability efficiency of computationally hard problems, capturing the true concept of algorithm in spite of providing a non–deterministic computing model.

The paper is organized as follows. In the next section, we give the definition of basic preliminary concepts about standard computational complexity. The third section presents the cellular framework where polynomial complexity classes will be defined. In Section 4 membrane systems able to produce an exponencial workspace in polynomial time are introduced. The paper ends by stressing some results concerning to complexity aspects in membrane computing, and by proposing several open questions of interest.

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