Weak Metrics on Configurations of a P System

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Abstract. The evolution of a P system generates a tree of computation potentially infinite where it is very difficult to set the degree of closeness between two configurations. The problem is specially hard if we want to quantify that proximity in order to make useful comparisons. In this paper we propose some weak metrics on configurations of a P system with a fixed structure of membranes and briefly discuss their advantages and drawbacks.

1 Introduction

In [2], a new model of computation within the framework of *Natural Computing* was introduced, called *P Systems*[1]. It starts from the assumption that the processes taking place in the compartmental structure of a living cell can be interpreted as computations.

Roughly speaking, a P system consists of a cell-like membrane structure, in the compartments of which one places multisets of objects which evolve according to given rules in a synchronous, parallel and non-deterministic manner.

The *membrane structure* of a P system is a hierarchical arrangement of membranes embedded in a *skin* membrane, the one which separates the system from its *environment.* A membrane without any membrane inside is called *elementary.* Each membrane defines a *region* (the closed space delimited by a mebrane and by the mebranes immediately inside it).

The *membrane structure* of a P system is used to enclose *computing cells* in The *membrane structure* is used to enclose *computing cells* in order to make them independent computing units. Also, a membrane serves as a communication channel between a given cell and other cells *adjacent* to it. The objects can pass through membranes and the membranes can be dissolved, divided or created.

A *configuration* is the instantaneous description of the current membrane structure and the multisets of objects associated with the membranes. In each time unit a transformation of a configuration of the system takes place by applying the rules of each region in a non-deterministic and maximally parallel manner. In this way, one gets transitions between the configurations of the system and a sequence of transitions is called a computation.

[1]A layman-oriented introduction can be found at [4] and further bibliography at [5].

Some times we need to know how different two configurations of P systems are. They can be different in many senses and the problem turns extremely hard when the configurations do not correspond to the same P systems. If we have three configurations $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$, is $\mathcal{C}_1$ *more different* from $\mathcal{C}_2$ than from $\mathcal{C}_3$? Is it possible to quantify this degree of similarity and give it an algebraic treatment? In this paper we study the differences among configurations and we propose a way to quantify the degree of difference.

We offer some solutions to the problem of finding appropriate metrics for P systems. We focus our attention only on finding metrics on the configurations of a P system with a fixed membrane structure. This involves a fixed alphabet and a fixed set of rules. In this case, two configurations may only differ in the multisets associated with the membranes. This difference can be measured between configurations not necessarily in the same branch of computation or in the same step.

We propose two models of defining metrics. The first one is based on the distance between regions. This gives us a very natural way of defining the distance according to the difference between multisets, but it does not consider the set of rules of the P system. The second model is based on the dependency graph associated with the rules of a P system and is based on the shortest paths in this directed graph.

The paper is organized as follows. Section 2 recalls some ideas about metric and weak metrics in a general way. In Section 3 two metrics on configuration of P systems based on the different multisets of regions are presented. In section 4 a new concept in P system theory is defined: the dependency graph of a P system. This dependency graph is used in Section 5 to define a weak metric on configurations. The paper finish with an example (Section 6) and some final remarks.

References

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[5] `http://psystems.disco.unimib.it/`